Trig Unit 1 Objective 3
April 16, 2010

• Evaluate inverse trig function for memorized values without a calculator.
• Evaluate inverse trig function for other values with a calculator.
Steps to find an inverse

1. Think inverse trig with Absolute value, e.g. $\sin^{-1}(-1/2)$, think $\sin^{-1}(1/2)$ first, and use it to find the reference angle.

2. Look at the sign ($\pm$) to determine which quadrants.

3. Use reference angle and quadrant(s) to get answer(s)

You may have more than one answer; but your calculator will always only give you one answer.

Ex 1: Solve for $\theta$: $\sin \theta = -\frac{1}{2}$ while $0^\circ \leq \theta < 360^\circ$

   1. $\sin^{-1}(-\frac{1}{2})$, $30^\circ$ is reference angle
   2. Quadrant: III, IV
   3. Answer: $210^\circ$, $330^\circ$

Ex 2: $\cos \theta = \frac{\sqrt{2}}{2}$ while $0 \leq \theta < 2\pi$

   1. The reference angle is $\pi/4$
   2. Quadrant: I, IV
   3. Answer: $\pi/4$, $7\pi/4$

You do: $\sin \theta = -\frac{\sqrt{3}}{2}$ while $0 \leq \theta < 2\pi$ (Ref angle: $\pi/3$, Quad: III or IV; Answer: $4\pi/3$ or $5\pi/3$)

$\cos \theta = -\frac{\sqrt{2}}{2}$ while $0^\circ \leq \theta < 360^\circ$ (Ref angle: $45^\circ$, Quad: II, III; Answer: $135^\circ$ or $225^\circ$)
Inverses of Axis Angles

• When the value of trig function is equal to 0, 1, or −1, think about North, South, East, and West.

• Ex 3
  – a. \( \sin \theta = 0 \), while \( 0^\circ \leq \theta < 360^\circ \)
  – b. \( \tan \theta = 0 \), while \( 0^\circ \leq \theta \leq 360^\circ \)

• You do: E
  
  (1) \( \cos \theta = 0 \), while \( 0 \leq \theta < 2\pi \)

  Answer: \( \theta = \pi/2 \) or \( 3\pi/2 \)

• (2) \( \sin \theta = −1 \), while \( 0^\circ \leq \theta < 360^\circ \)

  Answer: \( \theta = 270^\circ \)
More Examples

- **Ex 4:** $\cos \theta = -0.17$, while $0^\circ \leq \theta < 360^\circ$
  
  - Reference angle is: $80^\circ$
  
  - Quadrant: II and III
  
  - Answer: $100^\circ, 260^\circ$
  
  - Note: If you use calculator, calculator only gives you $100^\circ$

- **Ex 5:** $\sin \theta = -0.3$, while $0^\circ \leq \theta < 360^\circ$
  
  - Reference angle is: $17^\circ$
  
  - Quadrant: III and IV
  
  - Answer: $197^\circ, 343^\circ$
  
  - Note: If you use calculator, calculator only gives you $-17^\circ$ (which is $343^\circ$)
More Examples

• Ex 6: $\cos \theta = -0.3$ while $0 \leq \theta < 2\pi$
  Answer: 1.88 or 4.41

• You do: $\sin \theta = -0.78$ while $0 \leq \theta < 2\pi$
  Answer: 4.04 or 5.39

• Ex 7: $\tan \theta = 11.4$ while $0^\circ \leq \theta < 360^\circ$
  Answer: 85° or 265°

• Ex 8: $\sec \theta = 3.25$ while $0^\circ \leq \theta < 360^\circ$
  Answer: 72° or 288°
More Examples

• \( \sin \theta = -\frac{\sqrt{2}}{2} \) \( 0 \leq \theta < 2\pi \)  \( \text{Answer: } \frac{5\pi}{4} \text{ or } \frac{7\pi}{4} \)

• \( \cos \theta = \frac{\sqrt{3}}{2} \) \( 0 \leq \theta < 2\pi \)  \( \text{Answer: } \frac{\pi}{6} \text{ or } \frac{11\pi}{6} \)

• \( \sin \theta = 0 \) \( 0^\circ \leq \theta < 720^\circ \)  \( \text{Answer: } 0^\circ, 180^\circ, 360^\circ, 540^\circ \)

• \( \sin \theta = -0.358 \) \( 0 \leq \theta < 2\pi \)  \( \text{Answer: ref } \angle = 0.366, \theta = 3.51 \text{ or } 5.92 \)

• \( \cos \theta = -0.421 \) \( 0 \leq \theta < 2\pi \)  \( \text{Answer: ref } \angle = 1.14, \theta = 2.00 \text{ or } 4.28 \)

• \( \tan \theta = -21.12 \) \( 0 \leq \theta < 2\pi \)  \( \text{Answer: ref } \angle = 1.52, \theta = 1.62 \text{ or } 4.76 \)
The Unit Circle

- $\pi/2$ (90°) $(0, 1)$
- $2\pi/3$ (120°) $(-1/2, \sqrt{3}/2)$
- $\pi/3$ (60°) $(1/2, \sqrt{3}/2)$
- $\pi/4$ (45°) $(\sqrt{2}/2, \sqrt{2}/2)$
- $5\pi/6$ (150°) $(-\sqrt{3}/2, 1/2)$
- $\pi/6$ (30°) $(\sqrt{3}/2, 1/2)$
- $\pi$ (180°) $(-1, 0)$
- $7\pi/6$ (210°) $(-\sqrt{3}/2, -1/2)$
- $11\pi/6$ (330°) $(\sqrt{3}/2, -1/2)$
- $5\pi/4$ (225°) $(-\sqrt{2}/2, -\sqrt{2}/2)$
- $3\pi/2$ (270°) $(0, -1)$
- $4\pi/3$ (240°) $(-1, -\sqrt{3}/2)$
- $5\pi/3$ (300°) $(1/2, -\sqrt{3}/2)$
- $7\pi/4$ (315°) $(\sqrt{2}/2, -\sqrt{2}/2)$
- $0$ & $2\pi$ radians $0°$ & $360°$ $(1, 0)$
Positive: \( \sin, \csc \)
Negative: \( \cos, \tan, \sec, \csc, \cot \)