**Objective 0**: (review)
- Set up and solve trig word problems involves right triangles
- Fill in chart and evaluate sin, cos, tan of $30^\circ$ ($\pi/6$), $45^\circ$ ($\pi/4$), $60^\circ$ ($\pi/3$) without a calculator

Definition of trig function:
- $\sin \theta = \text{Opposite}/\text{Hypotenuse}$
- $\cos \theta = \text{Adjacent}/\text{Hypotenuse}$
- $\tan \theta = \text{Opposite}/\text{Hypotenuse}$

**SOH – CAH – TOA**

Ex.

\[
\begin{array}{c}
24 \\
5
\end{array}
\]

Ex. $\tan 2^\circ = 200/x$  $\Rightarrow x = 200/ \tan 2^\circ = 5727.3$

You need to memorize the following values:

<table>
<thead>
<tr>
<th></th>
<th>$30^\circ$ ($\pi/6$)</th>
<th>$45^\circ$ ($\pi/4$)</th>
<th>$60^\circ$ ($\pi/3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin$</td>
<td>$1/2$</td>
<td>$\sqrt{2}/2$</td>
<td>$\sqrt{3}/2$</td>
</tr>
<tr>
<td>$\cos$</td>
<td>$\sqrt{3}/2$</td>
<td>$\sqrt{2}/2$</td>
<td>$1/2$</td>
</tr>
<tr>
<td>$\tan$</td>
<td>$\sqrt{3}/3$</td>
<td>1</td>
<td>$\sqrt{3}$</td>
</tr>
</tbody>
</table>

The following are the definition of six trigonometric functions for a right triangle:

1. $\sin \theta = \text{opposite}/\text{hypotenuse}$  
   (sin is short for sine)
2. $\cos \theta = \text{adjacent}/\text{hypotenuse}$
   (cos is short for cosine)
3. $\tan \theta = \text{opposite}/\text{adjacent}$
   (tan is short for tangent)
4. $\csc \theta = \text{hypotenuse}/\text{opposite} = 1/\sin \theta$
   (csc is short for cosecant, and it is reciprocal of sine)
5. $\cot \theta = \text{adjacent}/\text{opposite} = 1/\tan \theta$
   (cot is short for cotangent, and it is reciprocal of tangent)
6. $\sec \theta = \text{hypotenuse}/\text{adjacent} = 1/\cos \theta$
   (sec is short for secant, and it is reciprocal of cosine)
Example of six trigonometric functions:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \sin \theta = \frac{3}{5} )</td>
</tr>
<tr>
<td>2.</td>
<td>( \cos \theta = \frac{4}{5} )</td>
</tr>
<tr>
<td>3.</td>
<td>( \tan \theta = \frac{3}{4} )</td>
</tr>
<tr>
<td>4.</td>
<td>( \csc \theta = \frac{5}{3} )</td>
</tr>
<tr>
<td>5.</td>
<td>( \sec \theta = \frac{5}{4} )</td>
</tr>
<tr>
<td>6.</td>
<td>( \cot \theta = \frac{4}{3} )</td>
</tr>
</tbody>
</table>

Activity in class: Color the unit circle with the following direction:

1. color the line red with the angle which is multiple of 90°
2. color the line blue with the angle which is multiple of 45°
3. color the line green with the angle which is multiple of 60°
4. color the line yellow with the angle which is multiple of 30°

4-13-10 Notes

**Trig Unit 1 Objectives**

- Draw angles in standard position and find co-terminal angles
- Find reference angles
- Convert between radians and degrees

**Angles in standard position** – An angle has initial side and terminal side, an angle is in standard position if the initial side is on x-axis.

An angle is **positive** if measured **counterclockwise** and an angle is **negative** if measured **clockwise**.

Example:

**Co-terminal angles**: Angles that have the same terminal side are called co-terminal angles.

Example: (1) 40° and –320° are co-terminal angles, you can find the co-terminal angles by adding ±360°

You do: find the co-terminal angles for 210°: Answers: −150°, 570°, 930°, −510°, ….etc

**Reference Angles**: The angles measured from the terminal side to the x-axis (always positive and less than 90°).

Example: Find the reference angle for \( \theta = 150° \);

Answer: 30° is the reference angle for 150°. (= 180° − 150°)

Example: Find the reference angle for \( \theta = 235° \);

Answer: 55° is the reference angle for 235°. (= 235° − 180°)

You do: find the reference angle for 320°. Answer: 40° is the reference angle for 320°.

**Measurement of an angle**: the measurement of an angle can be in degrees (denoted by “) or radians. Remember that a circle has 360° arc.

**Unit circle**: A circle is called a **unit circle** if the radius is one unit. The unit circle has circumference 2\( \pi \) which is the same as 360° arc length.

**Radian**: One radian is the angle measurement for \( \theta \) if the arc length is the same as the radius. The following figure is a unit circle with radius one and the arc length AB is 1

Conversion between degrees to radians: The ratio between degrees and radian is 180° : \( \pi \)

**Conversion from degrees to radians**: you multiply \( \pi/180° \)

**Example**: Convert 200° to radians

**Solution**: \( 200° \times \pi/180° = 10\pi/9 \)

**Conversion from radians to degrees**: you multiply 180°/\( \pi \)

**Example**: Convert \( 2\pi/3 \) to degrees

**Solution**: \( 2\pi/3 \times 180°/\pi = 120° \)

**Example**: Convert 2 radians to degree

**Solution**: \( 2 \times 180°/\pi = 360°/\pi \)

**Example**: Convert 330°t to radians

**Solution**: \( 330° \times \pi/180° = 11\pi/6 \)

Activity: fill in the unit circle with radians.
4-14-10 Activity:

Fill in the table values for sin, cos and tan with 30° ($\pi/6$), 45° ($\pi/4$), 60° ($\pi/3$), then complete the unit circle with the (x, y) coordinate.

<table>
<thead>
<tr>
<th>cos $\theta = x/1 = x$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin \theta = y/1 = y$</td>
<td>$\theta$</td>
</tr>
</tbody>
</table>

Thus the coordinate on the unit circle (x, y) is the same as (cos $\theta$, sin $\theta$)

4-16-10

Trig Unit 1 Objective 2

- Evaluate trig functions which are multiple of 30°, 45°, 60°, 90° in degrees, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$ in radians without a calculator.

Unit Circle is a circle with radius of one unit. As mentioned above, cos $\theta = x$, sin $\theta = y$.

The above figure indicate the sign of the trig functions:
A-S-T-C is an acronym: you can memorize as: All Students Take Calculus.

What does this acronym do for you? It helps you to decide the positive sign of the values of all six trigonometric functions.
A – all six trig functions have positive values in Quadrant I.
S – only sin and the reciprocal of sin (i.e. csc) have positive values in Quadrant II.
T – only tan and the reciprocal of tan (i.e. cot) have positive values in Quadrant III.
C – only cos and the reciprocal of cos (i.e. sec) have positive values in Quadrant IV.

**Steps to find the values of a trig function:**
1. Determine if the angle is an **axis angle** (the term **axis angle** was invented by Ms. Brice) which means the angle has initial and terminal sides on either x – axis or y – axis in standard position. (note that axis angle does not have reference angle, and reference angle is always less than 90°)
2. If it has a reference angle, then
   a. Use quadrant to find the sign “+” or “−”.
   b. Find the reference angle, use the reference angle to find the value and use the sign found in (a) to determine the value.
   c. Use the chart.

**Example:** Find sin (150°) = ?
**Solution:**
   a. 150° is in 2\(^{nd}\) quadrant, since we are looking for sine and sine is positive “+” (A-S-T-C).
   b. The reference angle is 30°, the value of sin (150°) is ½.
   c. Use the unit circle, the value is 1/2

**Example:** Find cos (150°) = ?
**Solution:**
   a. 150° is in 2\(^{nd}\) quadrant, since we are looking for cosine and cosine is negative “−” (A-S-T-C).
   b. The reference angle is 30°, the value of sin (150°) is \(\sqrt{3}/2\).
   c. Use the unit circle, the value is \(\sqrt{3}/2\)

3. If it is an axis angle, i.e. 0, \(\pi/2\), \(\pi\), 3\(\pi/2\), 2\(\pi\) or 0°, 90°, 180°, 270°, 360°, then use the unit circle values. Examples:
   a. sin \((\pi/2)\) = 1
   b. cos \((\pi/2)\) = 0
   c. sin \(3\pi/2\) = −1

**White board activities:**
1. Find the coordinates of the following angles on the unit circle: 150°, 225°, 300°
2. Use the unit circle chart to find the angles with the following coordinates: (−1/2, \(\sqrt{3}/2\)), (−\(\sqrt{2}/2\), −\(\sqrt{2}/2\)), (1/2, −\(\sqrt{3}/2\))

Find the angle “?” in the interval: 0° ≤ \(\theta\) < 360° or 0 ≤ \(\theta\) <2\(\pi\)

3. Find sin \(?° = \frac{1}{2}\) in degrees, i.e. find the angle \(\theta\) such that sin\(\theta\) = \(\frac{1}{2}\)   **Solution:** 30° or 150°
4. Find cos \(?° = −\sqrt{3}/2\) in radians. **Solution:** 5\(\pi/6\) or 7\(\pi/6\)
5. Find sin \(?° = −\sqrt{2}/2\) in degrees. **Solution:** 225° or 315°
6. Find cos \(?° = −1\). **Solution:** \(\pi\)