Chapter 2
Analysis of Graphs of Functions

Covered in this Chapter:
2.1 Graphs of Basic Functions and their Domain and Range. Odd, Even Functions, and their Symmetry.
2.2 Translations of Graphs: Vertical and Horizontal Shift, Stretching and Compressing, and Reflections.
2.3 Graphs of Absolute Value Functions. Equations, Inequalities, and Applications.
2.4 Piecewise Functions, Greatest Integer Function, and their Graphs.
2.5 Algebra of Functions and their Composition. Difference Quotient.
► Answers to all exercises.
2.1 Graphs of Functions

Definitions:

Continuous Functions:
A function is called continuous on its domain if it has no tears, gaps, or holes. That is if it can be drawn without lifting your pencil when you draw it on paper.

Increasing and Decreasing Functions:

A function $f$ is called increasing on an interval $I$ if
$$f(x_2) > f(x_1) \text{ whenever } x_2 > x_1 \text{ in } I$$

A function $f$ is called decreasing on an interval $I$ if
$$f(x_2) < f(x_1) \text{ whenever } x_2 > x_1 \text{ in } I$$

Constant Functions:

A function $f$ is called constant on an interval $I$ if
$$f(x_2) = f(x_1) \text{ whenever } x_2 > x_1 \text{ in } I$$

Symmetry of Functions:

Even Functions

A function $f$ is called an EVEN function if it satisfies the condition $f(-x) = f(x)$ for all $x$ in its domain. i.e. If the point $(x, y)$ is on the graph of $f(x)$, then the point $(-x, y)$ must be also on its graph.

An EVEN function is symmetric about the $y$-axis.
Examples of Even Functions

Odd Functions

A function $f$ is called an ODD function if it satisfies the condition $f(-x) = -f(x)$ for all $x$ in its domain.

i.e. If the point $(x, y)$ is on the graph of $f(x)$, then the point $(-x, -y)$ must be also on its graph.

An ODD function is symmetric about the origin.

Examples of Odd Functions

Tests for Symmetry:

Symmetry about the $y$-axis (Even functions):
Replace $x$ with $-x$ in $f(x)$. If $f(-x) = f(x)$, then the graph is symmetric about the $y$-axis.

Symmetry about the origin (Odd functions):
Replace $x$ with $-x$ in $f(x)$. If $f(-x) = -f(x)$, then the graph is symmetric about the origin.

Symmetry about the $x$-axis:
Replace $y$ with $-y$ in the equation. If you get the same equation, then the graph is symmetric about the $x$-axis. Remember that graphs symmetric about the $x$-axis are not functions. See the example below.
Example 1: Decide whether the function is odd, even or neither. \( f(x) = 2x^4 - 5x^2 + 10 \)

**Answer:** First replace \( x \) with \(-x\) in the given function.

\[
\begin{align*}
f(x) &= 2x^4 - 5x^2 + 10 \\
f(-x) &= 2(-x)^4 - 5(-x)^2 + 10 \\
f(-x) &= 2x^4 - 5x^2 + 10 = f(x) \\
\therefore f(-x) &= f(x) \\
\because \text{The given function is EVEN.}
\end{align*}
\]

(i.e. The function is symmetric about the \( y \)-axis).

Example 2: Decide whether the function is odd, even or neither. \( f(x) = 5x^3 - 4x - 3 \)

**Answer:** First replace \( x \) with \(-x\) in the given function.

\[
\begin{align*}
f(x) &= 5x^3 - 4x - 3 \\
f(-x) &= 5(-x)^3 - 4(-x) - 3 \\
f(-x) &= -5x^3 + 4x - 3 \\
f(-x) &= -(5x^3 - 4x + 3) \\
\therefore f(-x) &\neq f(x) \text{ nor } -f(x) \\
\therefore \text{The given function is NEITHER odd nor even.}
\end{align*}
\]

Example 3: Decide whether the function is odd, even or neither. \( f(x) = \frac{2x^3 - 5x}{4x^2 + 1} \)

**Answer:** First replace \( x \) with \(-x\) in the given function.

\[
\begin{align*}
f(x) &= \frac{2x^3 - 5x}{4x^2 + 1} \\
f(-x) &= \frac{2(-x)^3 - 5(-x)}{4(-x)^2 + 1} \\
f(-x) &= -\frac{2x^3 + 5x}{4x^2 + 1} \\
f(x) &= -\frac{2x^3 - 5x}{4x^2 + 1} \\
\therefore f(-x) &= -f(x) \\
\therefore \text{The given function is ODD.}
\end{align*}
\]

(i.e. The function is symmetric about the origin).
Basic Functions and their Graphs

**Identity Function**

- \( y = x \)
- Domain: \( \mathbb{R} \) or \( (-\infty, \infty) \)
- Range: \( \mathbb{R} \) or \( (-\infty, \infty) \)
- Odd Function
- Symmetric about the origin

**Absolute Value Function**

- \( y = |x| \)
- Domain: \( \mathbb{R} \) or \( (-\infty, \infty) \)
- Range: \( y \geq 0 \) or \( [0, \infty) \)
- Even function
- Symmetric about the \( y \)-axis

**Quadratic Function**

- \( y = x^2 \)
- Domain: \( \mathbb{R} \) or \( (-\infty, \infty) \)
- Range: \( y \geq 0 \) or \( [0, \infty) \)
- Even Function
- Symmetric about the \( y \)-axis

**Square Root Function**

- \( y = \sqrt{x} \)
- Domain: \( x \geq 0 \) or \( [0, \infty) \)
- Range: \( y \geq 0 \) or \( [0, \infty) \)
- Function is neither odd nor even

**Cubic Function**

- \( y = x^3 \)
- Domain: \( \mathbb{R} \) or \( (-\infty, \infty) \)
- Range: \( \mathbb{R} \) or \( (-\infty, \infty) \)
- Odd Function
- Symmetric about the origin
Cube Root Function

$$y = \sqrt[3]{x}$$
Domain: $$\mathbb{R}$$ or $$(-\infty, \infty)$$
Range: $$\mathbb{R}$$ or $$(-\infty, \infty)$$
Odd function
Symmetric about the origin

Exercises 2.1

In exercises 1 – 12, decide whether the function is odd, even, or neither and state whether its graph is symmetric about the origin, $$y$$-axis, or neither.

1) $$f(x) = 3x$$

2) $$f(x) = x^4 - |x|$$

3) $$f(x) = 2x^4 - 5x^2 + 3$$

4) $$f(x) = 4x^3 - 3x + 1$$

5) $$f(x) = 3x^5 - 2x^3 - 6x$$

6) $$f(x) = 2x^3 + 4x^2 - 5x + 1$$

7) $$f(x) = \frac{3x^2 - 1}{x^2 + 2}$$

8) $$f(x) = \frac{4x^3 - x}{x^2 - 3}$$

9) $$f(x) = \frac{2x^3 + 3x}{x + 1}$$

10) $$f(x) = \frac{2x^5 - 4x^3}{3x^3 + 5x}$$

11) $$f(x) = \frac{3x^4 + 5x^2 + 1}{2x^3 + 5x}$$

12) $$f(x) = \frac{x^2 + 4}{5x^3 - 2x}$$
In exercises 13 – 24, decide whether the graph is symmetric about the x-axis, y-axis, or the origin.
In this section, translations of graphs will be discussed. We will be using a screen produced by a TI 83+ showing an original function drawn in a thick line and some other functions that were produced from that original function. We should be able to come up with a conclusion from all these screens.

### Vertical Shifts

The above two screens show that if \( f(x) \) is a function and \( c \) is a positive number, then the graph of \( f(x) - c \) is exactly the same as the graph of \( f(x) \) but shifted down \( c \) units.

The above two screens show that if \( f(x) \) is a function and \( c \) is a positive number, then the graph of \( f(x) + c \) is exactly the same as the graph of \( f(x) \) but shifted up \( c \) units.

### Horizontal Shifts

The above two screens show that if \( f(x) \) is a function and \( c \) is a positive number, then the graph of \( f(x - c) \) is exactly the same as the graph of \( f(x) \) but shifted to the right \( c \) units.
The above two screens show that if $f(x)$ is a function and $c$ is a positive number, then the graph of $f(x + c)$ is exactly the same as the graph of $f(x)$ but shifted to the left $c$ units.

### Vertical Stretching

The above screens show that if $f(x)$ is a function and $c$ is a positive number greater than 1 ($c > 1$), then the graph of $[c f(x)]$ is exactly the same as the graph of $f(x)$ but vertically stretched by a factor of $c$.

### Vertical Shrinking

The above screens show that if $f(x)$ is a function and $c$ is a positive number such that $0 < c < 1$, then the graph of $[c f(x)]$ is exactly the same as the graph of $f(x)$ but vertically shrunk by a factor of $c$. 
Horizontal Shrinking

The above screens show that if \( f(x) \) is a function and \( c \) is a positive number greater than 1 (\( c > 1 \)), then the graph of \( f(cx) \) is exactly the same as the graph of \( f(x) \) but horizontally shrunk by a factor of \( c \).

Horizontal Stretching

The above screens show that if \( f(x) \) is a function and \( c \) is a positive number such that \( 0 < c < 1 \), then the graph of \( f(cx) \) is exactly the same as the graph of \( f(x) \) but horizontally stretched by a factor of \( c \).

Reflection about the \( x \)-axis

The above screens show that if \( f(x) \) is a function, then the graph of \([ -f(x) ]\) is exactly the same as the graph of \( f(x) \) but reflected about the \( x \)-axis.
The above screens show that if $f(x)$ is a function, then the graph of $f(-x)$ is exactly the same as the graph of $f(x)$ but reflected about the $y$-axis.

Now, that we know all operations that can be performed on a given graph, we should be able to answer the following examples.

**Example 1:** The graph of the function $f(x) = |x|$ is shifted to the right 3 units, then vertically stretched by a factor of 4, then reflected about the $y$-axis, and finally shifted down 5 units. Write the final translated function $h(x)$ and give its graph.

**Answer:**

$$f(x) = |x| \quad \xrightarrow{\text{Shifted to the right} \ 3 \ \text{units}} \quad |x - 3| \quad \xrightarrow{\text{Vertically stretched by} \ 4 \ \text{a factor of}} \quad 4|x - 3| \quad \xrightarrow{\text{Reflected about} \ the \ y-axis} \quad 4|-x - 3| \quad \xrightarrow{\text{Shifted down} \ 5 \ \text{units}} \quad 4|-x - 3| - 5 = h(x)$$
**Example 2:** The graph of the function \( f(x) = \sqrt{x} \) is shifted to the left 4 units, then vertically shrunk by a factor of \( \frac{1}{2} \), then reflected about the \( x \)-axis, and finally shifted up 3 units. Write the final translated function \( h(x) \) and give its graph.

**Answer:**

\[
f(x) = \sqrt{x} \quad \xrightarrow{\text{Shifted to the left 4 units}} \quad \sqrt{x + 4} \quad \xrightarrow{\text{Vertically shrunk by a factor of } \frac{1}{2}} \quad \frac{1}{2}\sqrt{x + 4} \quad \xrightarrow{\text{Reflected about the } x\text{-axis}} \quad -\frac{1}{2}\sqrt{x + 4} \quad \xrightarrow{\text{Shifted up 3 units}} \quad -\frac{1}{2}\sqrt{x + 4} + 3 = h(x)
\]

**Example 3:** The graph of the function \( f(x) = |x| \) is obtained by performing the following transformations on the graph of the function \( g(x) \) in the given order: shifted horizontally 5 units to the left, then vertically stretched by a factor of 4, reflected about the \( y \)-axis, and finally shifted downward 3 units. Find the rule of the function \( g(x) \).

**Solution:** We will show two methods to solve these types of problems.

**Method 1:** Following the exact steps as given and equating the final answer to \( f(x) \).

\[
g(x) \xrightarrow{\text{Shifted to the left 5 units}} g(x + 5) \xrightarrow{\text{Vertically stretched by a factor of } 4} 4g(x + 5) \xrightarrow{\text{Reflected about the } y\text{-axis}} 4g(-x + 5)
\]

\[
4g(-x + 5) \xrightarrow{\text{Shifted down 3 units}} 4g(-x + 5) - 3 = f(x) = |x|; \text{ solve for } g(x)
\]

\[
4g(-x + 5) - 3 = |x|; \text{ add 3 to both sides of the equation}
\]

\[
4g(-x + 5) = |x| + 3; \text{ multiply both sides of the equation by } \frac{1}{4}
\]
Method 2: The function \( g(x) \) is shifted to the left 5 units, vertically stretched by a factor of 4, reflected about the \( y \)-axis, and finally shifted down 3 units to obtain the function \( f(x) = |x| \).

Perform the opposite of each transformation in reverse order on \( f(x) \) to obtain the rule of the function \( g(x) \). The function \( f(x) = |x| \) is shifted up 3 units, reflected about the \( y \)-axis, vertically shrunk by a factor of 1/4, and finally shifted to the right 5 units.

- **Shift** \( f(x) = |x| \) **up** 3 units: \( f(x) + 3 = |x| + 3 = f_1(x) \)
- **Reflect** \( f_1(x) = |x| + 3 \) about the \( y \)-axis: \( f_1(-x) = |-x| + 3 = |x| + 3 = f_2(x) \)
- **Vertically shrink** \( f_2(x) = |x| + 3 \) by a factor of \( \frac{1}{4} \): \( \frac{1}{4} f_2(x) = \frac{1}{4} [ |x| + 3 ] = f_3(x) \)
- **Shift** \( f_3(x) = \frac{1}{4} [ |x| + 3 ] \) to the **right** 5 units: \( f_3(x - 5) = \frac{1}{4} [ |x - 5| + 3 ] = g(x) \)

\[ \therefore g(x) = \frac{1}{4} [ |x - 5| + 3 ] \]
Exercises 2.2

In exercises 1 – 8, write the function $g$ whose graph can be obtained from the graph of the function $f$ by performing the transformations in the given order. Graph the function $g$.

1) $f(x) = x^2 + 3$; shift the graph horizontally 2 units to the right and then vertically downward 5 units.

2) $f(x) = x^2 - 1$; shift the graph horizontally 3 units to the left and then vertically upward 4 units.

3) $f(x) = x^2 - 2x + 1$; reflect the graph about the $y$-axis, then shift it vertically downward 3 units.

4) $f(x) = x^2 + 4x + 4$; reflect the graph about the $y$-axis, then shift it vertically downward 1 unit.

5) $f(x) = x^2 - 2x + 1$; reflect the graph about the $x$-axis, then shift it vertically upward 2 units.

6) $f(x) = x^2 + 4x + 4$; reflect the graph about the $x$-axis, then shift it vertically upward 3 units.

7) $f(x) = \sqrt{x}$; shift the graph horizontally to the right 4 units, stretch it vertically by a factor of 3, reflect the graph about the $y$-axis, then shift it vertically upward 5 units.

8) $f(x) = \sqrt{x}$; shift the graph horizontally to the left 3 units, stretch it vertically by a factor of 2, reflect the graph about the $y$-axis, then shift it vertically downward 1 unit.

In exercises 9 – 14, use the graph of the function $g$ in the given figure to sketch the graph of the function $f$.

9) $f(x) = g(x) + 3$  
10) $f(x) = g(x) - 2$

11) $f(x) = 2g(x)$  
12) $f(x) = 0.5g(x)$

13) $f(x) = -g(x)$  
14) $f(x) = g(-x)$
In exercises 15 – 20, use the graph of the function $g$ in the given figure to sketch the graph of the function $f$.

15) $f(x) = g(x) + 1$
16) $f(x) = g(x) - 3$

17) $f(x) = 2g(x)$
18) $f(x) = 0.5g(x)$

19) $f(x) = -g(x) + 2$
20) $f(x) = g(-x) - 1$

21) The graph of the function $f(x) = x^2$ is obtained by performing the following transformations on the graph of the function $g(x)$ in the given order: shifted horizontally 2 units to the right, then vertically shrunk by a factor of 0.5, reflected about the $x$-axis, and finally shifted downward 3 units. Find the rule of the function $g(x)$.

22) The graph of the function $f(x) = x^2$ is obtained by performing the following transformations on the graph of the function $g(x)$ in the given order: shifted horizontally 3 units to the left, then vertically stretched by a factor of 2, reflected about the $y$-axis, and finally shifted upward 5 units. Find the rule of the function $g(x)$.

23) The graph of the function $f(x) = |x|$ is obtained by performing the following transformations on the graph of the function $g(x)$ in the given order: shifted horizontally 1 unit to the right, then vertically shrunk by a factor of 0.5, reflected about the $y$-axis, and finally shifted downward 2 units. Find the rule of the function $g(x)$.

24) The graph of the function $f(x) = \sqrt{x}$ is obtained by performing the following transformations on the graph of the function $g(x)$ in the given order: shifted horizontally 4 units to the left, then vertically shrunk by a factor of 0.25, reflected about the $x$-axis, and finally shifted downward 3 units. Find the rule of the function $g(x)$.
2.3 Absolute Equalities and Inequalities

**Absolute Value of a Function:**

\[
|f(x)| = \begin{cases} 
  f(x) & \text{if } f(x) \geq 0 \\
  -f(x) & \text{if } f(x) < 0
\end{cases}
\]

The above notation is simply saying that when the graph is zero or positive, leave it as it is. If the graph is negative then reflect it about the \(x\)-axis to make it positive.

**Example 1:** Given that \(f(x) = 0.1x^4 - 2x^2 + 3\), use your graphing calculator to graph \(y = |f(x)|\).

**Answer:** The first screen below shows the graph of the function \(f(x)\). The second screen shows both the graph of the function in thick lines and the graph of its absolute together. The last screen shows only the graph of the absolute of the function.

![Graphs of f(x) and |f(x)|](image)

**Absolute Equations:**

\[
|x| = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0
\end{cases}
\]

Absolute value of \(x\) measures the distance from \(x\) and zero. Remember that distance must be non-negative. Therefore \(|7| = 7\), \(|1| = 1\), \(|0| = 0\), \(|-5| = 5\), \(|-10| = 10\), and so on since the distance from any number to zero is the positive case of that number.

**Example 2:** Solve the absolute equation \(|x| = 3\).

**Answer:** The problem is simply saying that if the distance from \(x\) to zero is 3 units, what is \(x\)? We know that the distance must be non-negative. If we want to measure 3 units from zero, we have two ways to do so. Either we count 3 units to the right (this case we get +3) or we can count 3 units to the left (this case we get -3). This means we have two answers for \(x\), +3 and -3. The figure below shows how we got these two answers.

![Graph of |x| = 3](image)
To generalize the method of solving absolute equations, we have the following rules:

- If \(|ax + b| = c\) where \(c\) is a positive number \((c > 0)\) then, \(ax + b = c\) or \(ax + b = -c\)
- If \(|ax + b| = c\) where \(c\) is a negative number \((c < 0)\) then the equation has no solution
- If \(|ax + b| = 0\) then, \(ax + b = 0\)

**Example 3:** Solve the absolute equation \(|x + 3| = 5\).

**Answer:** First we need to apply the rule above and remove the absolute symbol by writing two equations.

\[
\begin{align*}
x + 3 &= 5 & x + 3 &= -5 \\
x &= 2 & x &= -8
\end{align*}
\]

\(\therefore x = \{2, -8\}\)

**Example 4:** Solve the absolute equation \(|2x - 7| - 6 = 5\).

**Answer:** We have to isolate the absolute expression before removing the absolute symbol. Therefore the equivalent equation is \(|2x - 7| = 11\). Now we should be able to write two equations and solve the problem.

\[
\begin{align*}
2x - 7 &= 11 & 2x - 7 &= -11 \\
2x &= 18 & 2x &= -4 \\
x &= 9 & x &= -2
\end{align*}
\]

\(\therefore x = \{-2, 9\}\)

**Example 5:** Solve the absolute equation \(|2x - 7| + 8 = 3\).

**Answer:** We have to isolate the absolute expression before removing the absolute symbol. Therefore the equivalent equation is \(|2x - 7| = -5\). But, we know that absolute value must be non-negative all the time. Therefore, the given absolute equation has no solution.

**Example 6:** Solve the absolute equation \(|2x - 10| = 0\).

**Answer:** In this type of problems when you see zero on one side of the equation, simply remove the absolute symbol and solve the equation as usual.

\[
\begin{align*}
|2x - 10| &= 0 \\
2x - 10 &= 0 \\
2x &= 10 \\
x &= 5
\end{align*}
\]
Solving absolute equations of the type \(|ax + b| = |cx + d|\)

If \(|ax + b| = |cx + d|\), then

\[ ax + b = cx + d \quad \text{or} \quad ax + b = -(cx + d) \]

**Example 7:** Solve the absolute equation \(|2x - 7| = |3x + 5|\).

**Answer:** Follow the above rule and write two equations to solve the problem.

\[
\begin{align*}
2x - 7 &= 3x + 5 \\
-x &= 12 \\
x &= -12
\end{align*}
\]

\[
\begin{align*}
2x - 7 &= -(3x + 5) \\
2x - 7 &= -3x - 5
\end{align*}
\]

\[
5x = 2, \ x = \frac{2}{5}
\]

\[
\therefore x = \left\{-\frac{12}{2}, \frac{2}{5}\right\}
\]

**Example 8:** Solve the absolute equation \(|3x + 5| = |3x - 2|\).

**Answer:** Write two equations and solve the problem.

\[
\begin{align*}
3x + 5 &= 3x - 2 \\
3x - 3x &= -2 - 5 \\
0 &= -7 \\
\text{No Solution}
\end{align*}
\]

\[
\begin{align*}
3x + 5 &= -(3x - 2) \\
3x + 5 &= -3x + 2
\end{align*}
\]

\[
6x = -3, \ x = -\frac{1}{2}
\]

\[
\therefore x = \left\{-\frac{1}{2}\right\}
\]

**Note:** In the last example, don’t write “the answer is no solution or -1/2”. Just write -1/2

**Solving Absolute Inequalities**

- Solving inequalities of the form \(|ax + b| \leq c|\)

\[
\text{If} \ |ax + b| \leq c \quad \text{where} \ c \ \text{is a positive number} \ (c > 0), \ \text{then}
\]

\[ ax + b \leq c \quad \text{and} \quad ax + b \geq -c \]

If we write the above as continued inequalities, we can write the inequality as follows:

\[ -c \leq ax + b \leq c \]

\[
\text{If} \ |ax + b| \leq c \quad \text{where} \ c \ \text{is a negative number} \ (c < 0), \ \text{then}
\]

the inequality has no solution

\[ \text{If} \ |ax + b| \leq 0, \ \text{then} \ ax + b = 0 \]

**Note:** In the above rule, due to space limitation, we used the symbol of inequality less than or equal (\(\leq\)). The less than symbol (\(<\)) also applies to this rule.
The figure below shows why we call the above inequality the **AND** case. Notice that the interval of the solution is one continuous interval. If \( x \) is in the interval \((-3, 3)\), we can see that \( x \) is less than 3 **AND** \( x \) is greater than -3.

**Example 9:** Solve the absolute inequality. 

\[ |x + 3| \leq 5 \]

**Answer:** First we need to apply the rule above and remove the absolute symbol by writing continued inequalities as follows:

\[
\begin{align*}
-5 &\leq x + 3 \leq 5 \\
-5 - 3 &\leq x + 3 - 3 \leq 5 - 3 \\
-8 &\leq x \leq 2
\end{align*}
\]

Set Notation: \( \{x \mid -8 \leq x \leq 2\} \)  
Interval Notation: \([-8, 2]\)

**Example 10:** Solve the absolute inequality. 

\[ |2x - 7| - 6 < 5 \]

**Answer:** We have to isolate the absolute expression before removing the absolute symbol. Therefore the equivalent inequality is 

\[ |2x - 7| < 11 \]

Now we should be able to write the inequality as continued inequalities and solve the problem.

\[
\begin{align*}
-11 &< 2x - 7 < 11 \\
-11 + 7 &< 2x - 7 + 7 < 11 + 7 \\
-4 &< 2x < 18 \\
-2 &< x < 9
\end{align*}
\]

Set Notation: \( \{x \mid -2 < x < 9\} \)  
Interval Notation: \((-2, 9]\)

**Example 11:** Solve the absolute inequality. 

\[ |2x - 7| + 8 < 3 \]

**Answer:** We have to isolate the absolute expression before removing the absolute symbol. Therefore the equivalent inequality is 

\[ |2x - 7| < -5 \]

But we know that absolute value must be non-negative all the time. Therefore, the given absolute inequality has no solution.

**Example 12:** Solve the absolute inequality. 

\[ |2x - 10| \leq 0 \]

**Answer:** In these types of problems when you see that the inequality expression is less than or equal to zero, remove the absolute symbol, replace the inequality symbol with an equal sign and solve the equation.

\[
\begin{align*}
|2x - 10| &= 0 \\
2x - 10 &= 0 \\
2x &= 10 \\
x &= 5
\end{align*}
\]
Solving inequalities of the form $|ax + b| \geq c$

- If $|ax + b| \geq c$ where $c$ is a positive number ($c > 0$), then
  
  - $ax + b \geq c$ or $ax + b \leq -c$

- If $|ax + b| \geq c$ where $c$ is a non-positive number ($c \leq 0$), then
  
  - the solution is the set of all real numbers $\mathbb{R}$

- If $|ax + b| > 0$, then the solution is all real numbers except $x = -\frac{b}{a}$

  - that is: $(-\infty, -\frac{b}{a}) \cup (-\frac{b}{a}, \infty)$

**Note:** In the above rule, due to space limitation, we used the symbol of inequality greater than or equal ($\geq$). The greater than symbol ($>$) also applies to this rule.

The figure below shows why we call the above inequality the **OR** case. Notice that the interval of the solution is two separate intervals. If $x$ is in the interval $(-\infty, -3)$ then $x$ is less than -3. If $x$ is in the interval $(3, \infty)$ then $x$ is greater than 3. Since $x$ cannot be in two places at the same time, we have to say that $x$ is less than -3 **OR** $x$ is greater than 3.

**Example 13:** Solve the absolute inequality $|x + 3| \geq 5$

**Answer:** First we need to apply the rule above and remove the absolute symbol by writing the absolute inequality as two separate inequalities.

- \[ x + 3 \leq -5 \]
- \[ x + 3 - 3 \leq -5 - 3 \]
- \[ x \leq -8 \]
- \[ x + 3 \geq 5 \]
- \[ x + 3 - 3 \geq 5 - 3 \]
- \[ x \geq 2 \]

  - Set Notation: $\{x \mid x \leq -8 \text{ or } x \geq 2\}$
  - Interval Notation: $(-\infty, -8] \cup [2, \infty)$

**Example 14:** Solve the absolute inequality $|2x - 7| + 8 > 3$

**Answer:** We have to isolate the absolute expression before removing the absolute symbol Therefore the equivalent equation is $|2x - 7| > -5$.
But, we know that absolute value must be non-negative all the time. Therefore, the solution to the given absolute inequality is the set of all real numbers.

**Example 15:** Solve the absolute inequality $|2x - 7| - 6 > 5$

**Answer:** We have to isolate the absolute expression before removing the absolute symbol Therefore the equivalent inequality is $|2x - 7| > 11$. Now we should be able to write the inequality as two separate inequalities and solve the problem.

<table>
<thead>
<tr>
<th>$2x - 7 &lt; -11$</th>
<th>$2x - 7 &gt; 11$</th>
<th>Set Notation: ${x \mid x &lt; -2 \text{ or } x &gt; 9}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x - 7 + 7 &lt; -11 + 7$</td>
<td>$2x - 7 + 7 &gt; 11 + 7$</td>
<td>Interval Notation: $(-\infty, -2) \cup (9, \infty)$</td>
</tr>
<tr>
<td>$2x &lt; -4$</td>
<td>$2x &gt; 18$</td>
<td></td>
</tr>
<tr>
<td>$x &lt; -2$</td>
<td>$x &gt; 9$</td>
<td></td>
</tr>
</tbody>
</table>

**Example 16:** Solve the absolute inequality $|2x - 10| > 0$

**Answer:** In these types of problems when you see that the inequality expression is greater than zero, remove the absolute symbol, replace the inequality symbol with an equal sign and solve the equation.

\[
|2x - 10| = 0 \\
2x - 10 = 0 \\
2x = 10 \\
x = 5
\]

.: The solution is all real numbers except 5 or $\mathbb{R}$ except 5.

Interval notation: $(-\infty, 5) \cup (5, \infty)$.

Now, that we have discussed all types of absolute equalities and inequalities, we should be able to solve an example with absolute equality and its related inequalities.

**Example 17:** Solve the absolute equality and its given related inequalities.

a) $|2x - 3| = 7$  

**Answer:**

<table>
<thead>
<tr>
<th>$2x - 3 = 7$</th>
<th>$2x - 3 = -7$</th>
<th>$2x = 10 \quad , \quad 2x = -4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x = -3 \quad , \quad 2x = -7 + 3$</td>
<td>$2x = 7 + 3 \quad , \quad 2x = -4 \quad , \quad 2x = 10$</td>
<td>$x = -2 \quad , \quad x = 5$</td>
</tr>
</tbody>
</table>

$\{x \mid -2, 5\}$

b) $|2x - 3| \leq 7$

\[
-7 \leq 2x - 3 \leq 7 \\
-7 + 3 \leq 2x \leq 7 + 3 \\
-4 \leq 2x \leq 10 \\
-2 \leq x \leq 5
\]

Interval Notation: $[-2, 5]$, Set Notation: $\{x \mid -2 \leq x \leq 5\}$

c) $|2x - 3| \geq 7$

\[
2x - 3 \leq -7 \quad , \quad 2x - 3 \geq 7 \\
2x \leq -7 + 3 \quad , \quad 2x \geq 7 + 3 \\
2x \leq -4 \quad , \quad 2x \geq 10 \\
x \leq -2 \quad ; \quad x \geq 5
\]

Interval Notation: $(-\infty, -2] \cup [5, \infty)$, Set Notation: $\{x \mid x \leq -2 \text{ or } x \geq 5\}$. 
Example 18: Use your graphing calculator to solve the absolute equality and its related inequalities.

a) \(|x+1|+|x-6|=10\)  
b) \(|x+1|+|x-6|\leq10\)  
c) \(|x+1|+|x-6|\geq10\)

Answer: First enter the left side of the equality as \(Y_1\) and the right hand side as \(Y_2\).
Exercises 2.3

In exercises 1 – 16, solve the absolute equations.

1) \(|x - 5| = 4\)  
2) \(|x - 2| = 3\)  
3) \(|2x + 3| = 5\)  
4) \(|3x + 1| = 7\)  
5) \(|2x - 7| + 2 = 8\)  
6) \(|3x + 4| + 1 = 3\)  
7) \(|5x + 2| + 6 = 10\)  
8) \(|4x - 5| + 4 = 3\)  
9) \(|3x - 2| + 5 = 2\)  
10) \(|5x - 1| + 9 = 5\)  
11) \(|2x - 6| + 5 = 5\)  
12) \(|3x - 9| + 7 = 7\)  
13) \(|x - 3| = |2x + 5|\)  
14) \(|3x + 5| = |5x - 2|\)  
15) \(|2x + 1| = |2x - 3|\)  
16) \(|5x - 4| = |5x + 1|\)

In exercises 17 – 40, solve the absolute inequalities.

17) \(|x - 4| \leq 3\)  
18) \(|x + 1| \leq 8\)  
19) \(|2x - 3| < 5\)  
20) \(|4x - 3| < 1\)  
21) \(|2x + 4| - 3 \leq 5\)  
22) \(|3x - 2| - 4 \leq 6\)  
23) \(|2x - 10| + 5 < 5\)  
24) \(|2x - 6| + 3 < 3\)  
25) \(|5x - 2| + 6 \leq 2\)  
26) \(|6x + 5| + 7 \leq 2\)  
27) \(|2x - 10| + 1 \leq 1\)  
28) \(|2x - 6| + 4 \leq 4\)  
29) \(|x - 2| \geq 5\)  
30) \(|x + 3| \geq 4\)  
31) \(|3x + 2| > 5\)  
32) \(|5x - 4| > 1\)  
33) \(|2x + 3| - 4 \geq 2\)  
34) \(|3x + 5| - 2 \geq 8\)  
35) \(|2x - 10| + 11 > 11\)  
36) \(|2x - 6| + 7 > 7\)  
37) \(|5x - 2| + 6 \geq 2\)  
38) \(|6x + 5| + 7 \geq 2\)  
39) \(|2x - 10| + 9 \geq 9\)  
40) \(|2x - 6| + 5 \geq 5\)

In exercises 41 – 46, solve the absolute equation and its related inequalities.

41. a) \(|x| + |x + 1| = 3\)  
b) \(|x| + |x + 1| \geq 3\)  
c) \(|x| + |x + 1| \leq 3\)  
42. a) \(|x| + |x + 2| = 6\)  
b) \(|x| + |x + 2| \geq 6\)  
c) \(|x| + |x + 2| \leq 6\)  
43. a) \(|x - 1| + |x + 3| = 9\)  
b) \(|x - 1| + |x + 3| > 9\)  
c) \(|x - 1| + |x + 3| < 9\)  
44. a) \(|x + 4| + |x - 3| = 5\)  
b) \(|x + 4| + |x - 3| \geq 5\)  
c) \(|x + 4| + |x - 3| \leq 5\)  
45. a) \(|x + 5| + |x - 3| = 11\)  
b) \(|x + 5| + |x - 3| \geq 11\)  
c) \(|x + 5| + |x - 3| \leq 11\)  
46. a) \(|x - 4| + |x + 2| = 5\)  
b) \(|x - 4| + |x + 2| > 5\)  
c) \(|x - 4| + |x + 2| < 5\)
2.4 Piecewise Functions

Definition: A piecewise function is a defined function that consists of several functions (i.e. the graph of a piecewise function consists of several graphs. Each one of these graphs is also a function). Graphs of parts of a piecewise function usually have one or more of the following: holes, sharp corners, and jumps.

Example 1: Graph the following piecewise function and evaluate it at the given values.

\[
f(x) = \begin{cases} 
  x^2, & \text{if } x \leq 0 \\
  2x - 5, & \text{if } x > 0
\end{cases}
\]

Answer: To graph the given piecewise function, we need to graph each part separately. Be sure to graph each part within its defined domain. You can see that the first part which is a parabola is defined only over the interval \((-\infty, 0]\) and the point zero is included (closed circle). The second part represents a line. The line is defined only on the interval \((0, \infty)\) where point zero is excluded (open circle).

Now, press \(\text{TRACE}\) and enter the values for \(x\) at which you want to evaluate the function \(f\).

\[
\begin{align*}
  f(3) &= (3)^2 - 5 = 1, \text{ use the second function since 3 is in its domain only. } \\
  f(-2) &= (-2)^2 = 4, \text{ use the first function since -2 is in its domain only } \\
  f(0) &= (0)^2 = 0, \text{ use the first function since 0 is in its domain only }
\end{align*}
\]

Greatest Integer Function (Step Function):

Notation: \( f(x) = \lfloor x \rfloor \); which means the greatest integer that is less than or equal to \(x\). Equality will occur only when \(x\) is an integer. If not, always round down.

Example 2: Evaluate the following: \( f(x) = \{[3.4], [5], [1.2], [0], [-1.45], [-7]\} \)

Answer: Since the value of the function is equal to \(x\) if \(x\) is an integer, we need to round down if \(x\) is not an integer, \(\therefore f(x) = \{3, 5, 1, 0, -2, -7\} \)
Graphing Calculator Solution of Example 2: Press \[ \text{MATH NUM 5:int(} \]

Graph of the greatest integer function \( f(x) = [x] \):

Domain: The set of all real numbers \( \mathbb{R} \) or \( (-\infty, \infty) \)

Range: The set of all integers

Using a graphing calculator to graph \( f(x) = [x] \):

To find "int", press \[ \text{MATH NUM 5:int(} \]

Don’t be deceived by the graph above. This is a lack of technology. All the vertical segments are not part of the graph. If you want to see a better graph, you need to change the MODE in your TI from Connected to Dot. To do this, press MODE, move the cursor down to Dot and press ENTER to highlight Dot.

If you do this you’ll get the screen below. Now you can see why we call this function “Step Function".
Example 3: Graph the function \( f(x) = \lceil x - 1 \rceil \), evaluate \( f(-1.3), f(-3), f(2.3), f(4), \) and \( f(7.1) \).

Answer: Since the value of the function is equal to \( x \) if \( x \) is an integer, round down if \( x \) is not an integer.

\[
\begin{align*}
  f(-1.3) &= \lceil -1.3 - 1 \rceil = \lceil -2.3 \rceil = -3 \\
  f(-3) &= \lceil -3 - 1 \rceil = \lceil -4 \rceil = -4 \\
  f(2.3) &= \lceil 2.3 - 1 \rceil = \lceil 1.3 \rceil = 1 \\
  f(4) &= \lceil 4 - 1 \rceil = \lceil 3 \rceil = 3 \\
  f(7.1) &= \lceil 7.1 - 1 \rceil = \lceil 6.1 \rceil = 6
\end{align*}
\]

Note: Due to the TI Graphing calculator limitation, we are going to assign a plus sign to each closed circle and a square to each open circle on the graph of the function generated by the TI.

Using the Table Feature:

Press \textit{2nd TBLSET}, highlight \textit{Ask} to the right of \textit{Indpnt:}, and highlight \textit{Auto} to the right of \textit{Depend:}.

Press \textit{2nd TABLE}, enter the value for \( x \) at which you want to evaluate the function, and then press \textit{ENTER}.

From the last screen, \( f(-1.3) = -3, f(-3) = -4, f(2.3) = 1, f(4) = 3, \) and \( f(7.1) = 6 \).
Exercises 2.4

In exercises 1 – 10, graph the function and evaluate it at the given values.

1) \( f(x) = \begin{cases} x - 3 & \text{if } x < -2 \\ -x + 5 & \text{if } x \geq -2 \end{cases} \), evaluate \( f(4) \), \( f(-5) \), and \( f(2) \)

2) \( f(x) = \begin{cases} 2x + 3 & \text{if } x \leq -1 \\ 1 - 4x & \text{if } x > -1 \end{cases} \), evaluate \( f(4) \), \( f(-5) \), and \( f(-1) \)

3) \( f(x) = \begin{cases} 1 - 2x & \text{if } x < 1 \\ 2 + 3x & \text{if } x \geq 1 \end{cases} \), evaluate \( f(0) \), \( f(-1) \), \( f(1) \), and \( f(2) \)

4) \( f(x) = \begin{cases} 3x + 2 & \text{if } x < 2 \\ 2x - 3 & \text{if } x \geq 2 \end{cases} \), evaluate \( f(0) \), \( f(-2) \), \( f(2) \), and \( f(3) \)

5) \( f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 4x + 3 & \text{if } x \geq 0 \end{cases} \), evaluate \( f(0) \), \( f(-1) \), and \( f(1) \)

6) \( f(x) = \begin{cases} 2x - 3 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases} \), evaluate \( f(0) \), \( f(-1) \), and \( f(1) \)

7) \( f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 3x - 5 & \text{if } x > 1 \end{cases} \), evaluate \( f(3) \), \( f(1) \), and \( f(-2) \)

8) \( f(x) = \begin{cases} 2x - 3 & \text{if } x \leq -2 \\ x^2 & \text{if } x > -2 \end{cases} \), evaluate \( f(0) \), \( f(-2) \), and \( f(-3) \)

9) \( f(x) = \begin{cases} 2x + 1 & \text{if } x \leq -2 \\ x^2 & \text{if } -2 < x < 1 \\ 3x - 1 & \text{if } x \geq 1 \end{cases} \), evaluate \( f(0) \), \( f(-2) \), \( f(1) \), \( f(-3) \), and \( f(4) \)

10) \( f(x) = \begin{cases} x - 5 & \text{if } x \leq -1 \\ x^2 & \text{if } -1 < x < 2 \\ 9 - x & \text{if } x \geq 2 \end{cases} \), evaluate \( f(0) \), \( f(-1) \), \( f(2) \), \( f(-3) \), and \( f(4) \)
In exercises 11 – 18, graph the function and evaluate it at the given values.

11) \( f(x) = \lfloor x - 2 \rfloor \), evaluate \( f(-2.3), f(-5), f(4.1), f(3), \) and \( f(5.1) \)

12) \( f(x) = \lfloor x - 1 \rfloor \), evaluate \( f(-1.3), f(-3), f(2.3), f(4), \) and \( f(7.1) \)

13) \( f(x) = \lfloor x + 1 \rfloor \), evaluate \( f(-1.2), f(-5.3), f(0), f(2.5), \) and \( f(7) \)

14) \( f(x) = \lfloor x + 2 \rfloor \), evaluate \( f(-3.2), f(-4), f(-0.1), f(6), \) and \( f(8.1) \)

15) \( f(x) = \lfloor x + 3 \rfloor + 1 \), evaluate \( f(-1.2), f(-2), f(-2.1), f(3), \) and \( f(4.1) \)

16) \( f(x) = \lfloor x - 3 \rfloor - 1 \), evaluate \( f(-4.2), f(-1), f(0), f(5), \) and \( f(3.1) \)

17) \( f(x) = \lfloor x \rfloor + 2 \), evaluate \( f(-3.2), f(-1), f(0), f(2), \) and \( f(7.2) \)

18) \( f(x) = \lfloor x \rfloor - 3 \), evaluate \( f(-5.2), f(-2), f(0), f(2.3), \) and \( f(5) \)
2.5 Algebra of Functions and their Composites

Two functions \( f \) and \( g \) can be combined to form a new function \( h \) exactly the same way we add, subtract, multiply, and divide real numbers.

\[
\begin{align*}
\text{Algebra of Functions} \\
&\text{Let } f \text{ and } g \text{ be two functions with domains } A \text{ and } B \text{ respectively then:} \\
h_1(x) &= (f + g)(x) = f(x) + g(x) \quad \text{domain} = \{x \mid x \in A \cap B\} \\
h_2(x) &= (f - g)(x) = f(x) - g(x) \quad \text{domain} = \{x \mid x \in A \cap B\} \\
h_3(x) &= (fg)(x) = f(x) \times g(x) \quad \text{domain} = \{x \mid x \in A \cap B\} \\
h_4(x) &= \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \text{domain} = \{x \mid x \in A \cap B \text{ and } g(x) \neq 0\} \\
\end{align*}
\]

The Graph of the function \( h(x) = (f + g)(x) \) can be obtained from the graph of \( f \) and \( g \) by adding the corresponding \( y \)-coordinates as shown on the figure below.

Similarly, we can find the difference, product, and division of \( f \) and \( g \). In case of division, remember that \( g(x) \neq 0 \).

Example 1: If \( f(x) = 2x - 5 \) and \( g(x) = x + 3 \), find \((f + g)(x), (f - g)(x), (fg)(x), \text{ the } \left(\frac{f}{g}\right)(x)\).

Also evaluate \((f + g)(1)\) and \(\left(\frac{f}{g}\right)(2)\).

Answer:

\[
\begin{align*}
(f + g)(x) &= f(x) + g(x) = (2x - 5) + (x + 3) = 3x - 2 \\
(f - g)(x) &= f(x) - g(x) = (2x - 5) - (x + 3) = 2x - 5 - x - 3 = x - 8 \\
(fg)(x) &= f(x)g(x) = (2x - 5)(x + 3) = 2x^2 + x - 15 \\
\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)} = \frac{2x - 5}{x + 3}, \; x \neq -3 \\

\text{To evaluate } (f + g)(1), \text{ and } \left(\frac{f}{g}\right)(2): \\
(f + g)(1) &= 3(1) - 2 = 1 \\
\left(\frac{f}{g}\right)(2) &= \frac{2(2) - 5}{2 + 3} = -\frac{1}{5}
\end{align*}
\]
Composition of Functions

**Notation:** \( (f \circ g)(x) = f(g(x)) \)

The above can be read in many ways. It can be read as “\( f \) composed with \( g \) at \( x \)” or simply “\( f \) composed with \( g \)” or “the composition of \( f \) and \( g \)”.

The Composite Function \( f \circ g \) is defined by

\[
(f \circ g)(x) = f(g(x))
\]

Its domain is all numbers \( x \) in the domain of \( g(x) \)
such that \( g(x) \) is in the domain of \( f \)

---

**Example 2:** \( f(x) = 3x - 5 \), \( g(x) = 2x + 3 \), find \( (f \circ g)(x) \) and \( (g \circ f)(x) \)

**Answer:**

\[
(f \circ g)(x) = f(g(x)) = 3(g(x)) - 5 = 3(2x + 3) - 5 = 6x + 4
\]

\[
(g \circ f)(x) = g(f(x)) = 2(f(x)) + 3 = 2(3x - 5) + 3 = 6x - 7
\]

**Note:** In general \( (f \circ g)(x) \neq (g \circ f)(x) \)

**Example 3:** \( f(x) = 2x - 7 \), \( g(x) = 5x + 1 \), find \( (f \circ g)(3) \)

**Answer:** We can use two different methods to solve this problem.

**First method:** Find \( (f \circ g)(x) \) and then substitute 3 for \( x \)

\[
(f \circ g)(x) = f(g(x)) = 2(g(x)) - 7 = 2(5x + 1) - 7 = 10x - 5
\]

\[
(f \circ g)(3) = 10(3) - 5 = 25
\]

**Second method:** \( \because (f \circ g)(3) = f(g(3)) \), evaluate \( g(3) \) and substitute the result into \( f \)

\[
g(3) = 5(3) + 1 = 15 + 1 = 16
\]

\[
(f \circ g)(3) = f(g(3)) = f(16) = 2(16) - 7 = 32 - 7 = 25
\]
The Difference Quotient

The difference quotient expression is very important in the study of calculus. Looking at the graph below, the difference quotient is the slope of the line PQ which is called the secant line.

From the graph, we see that the slope of secant line PQ is \( m_{PQ} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{Rise}}{\text{Run}} = \frac{f(x+h) - f(x)}{h}. \)

Example 4: Let \( f(x) = x^2 - 3x + 5 \), find the difference quotient and simplify your answer

Answer: For clarity, I’ll solve this example step by step. Some of the steps can be eliminated.

\[
\begin{align*}
  f(x) &= x^2 - 3x + 5 \\
  f(x+h) &= (x+h)^2 - 3(x+h) + 5 = x^2 + 2xh + h^2 - 3x - 3h + 5 \\
  f(x+h) - f(x) &= (x^2 + 2xh + h^2 - 3x - 3h + 5) - (x^2 - 3x + 5) \\
  &= 2xh + h^2 - 3h \\
  &= h(2x + h - 3) \\
  m &= \frac{f(x+h) - f(x)}{h} = \frac{h(2x + h - 3)}{h} = 2x + h - 3
\end{align*}
\]

Note 1: The simplified form of the numerator \( f(x+h) - f(x) \) should have \( h \) as a common factor.

Note 2: Review your work if all the terms of the original functions \( f(x) \) don’t cancel in the expression of the difference quotient.
Exercises 2.5

Let \( f(x) = 2x - 3 \) and \( g(x) = 4x + 5 \). Find the following.

1. \((f + g)(x)\)
2. \((f - g)(x)\)
3. \((f \cdot g)(x)\)
4. \((\frac{f}{g})(x)\)
5. \((f + g)(1)\)

6. \((f \cdot g)(2)\)
7. \((f - g)(-2)\)
8. \((g - f)(0)\)
9. \((\frac{f}{g})(3)\)
10. \((\frac{g}{f})(-1)\)

Let \( f(x) = 2x + 4 \) and \( g(x) = x - 3 \). Find the following.

11. \((f + g)(x)\)
12. \((f - g)(x)\)
13. \((f \cdot g)(x)\)
14. \((\frac{f}{g})(x)\)
15. \((f + g)(2)\)

16. \((f \cdot g)(-1)\)
17. \((f - g)(-3)\)
18. \((g - f)(0)\)
19. \((\frac{f}{g})(1)\)
20. \((\frac{g}{f})(4)\)

Let \( f(x) = 4x - 1 \) and \( g(x) = 2x + 3 \). Find the following.

21. \((f \circ g)(x)\)
22. \((g \circ f)(x)\)
23. \((f \circ f)(x)\)
24. \((g \circ g)(x)\)

25. \((f \circ g)(1)\)
26. \((g \circ f)(-2)\)
27. \((f \circ f)(-1)\)
28. \((g \circ g)(0)\)

Let \( f(x) = x^2 - 3 \) and \( g(x) = 3x - 1 \). Find the following.

29. \((f \circ g)(x)\)
30. \((g \circ f)(x)\)
31. \((f \circ f)(x)\)
32. \((g \circ g)(x)\)

33. \((f \circ g)(2)\)
34. \((g \circ f)(0)\)
35. \((f \circ f)(-2)\)
36. \((g \circ g)(1)\)

In exercises 37 – 44, find the difference quotient of each given function and simplify answer completely.

37. \( f(x) = 2x + 5 \)

38. \( f(x) = 3x - 4 \)

39. \( f'(x) = 5x + 4 \)

40. \( f(x) = 3x + 7 \)

41. \( f(x) = x^2 - 3x + 1 \)

42. \( f(x) = x^2 + 2x - 1 \)

43. \( f(x) = x^2 - 2x + 3 \)

44. \( f(x) = x^2 + 4x - 1 \)
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Answers - Exercises 2.1

22. All 23. x-axis 24. y-axis

Answers - Exercises 2.2

1. \( g(x) = (x-2)^2 - 2 \)
2. \( g(x) = (x+3)^2 + 3 \)
3. \( g(x) = x^2 + 2x - 2 \)
4. \( g(x) = x^2 - 4x + 3 \)
5. \( g(x) = -x^2 + 2x + 1 \)
6. \( g(x) = -x^2 - 4x - 1 \)
7. \( g(x) = 5 + 3\sqrt{-x-4} \)
8. \( g(x) = -1 + 2\sqrt{-x+3} \)

9. 
10. 
11. 
12. 
13. 
14. 
15. 
16. 

21. \( g(x) = -2(x+2)^2 - 6 \)
22. \( g(x) = \frac{1}{2}(x-3)^2 - \frac{5}{2} \)
23. \( g(x) = 2|x+1| + 4 \)
24. \( g(x) = -4\sqrt{x-4} - 12 \)
Answers - Exercises 2.3  

1. \( x = \{1, 9\} \)  
2. \( x = \{-1, 5\} \)  
3. \( x = \{-4, 1\} \)  
4. \( x = \left\{ \frac{8}{3}, 2 \right\} \)  
5. \( x = \left\{ \frac{1}{2}, \frac{13}{2} \right\} \)  
6. \( x = \left\{ -2, \frac{-2}{3} \right\} \)  
7. \( x = \left\{ \frac{-6}{5}, \frac{2}{5} \right\} \)  
8. \( \phi \)  
9. \( \phi \)  
10. \( \phi \)  
11. \( x = \{3\} \)  
12. \( x = \{3\} \)  
13. \( x = \left\{ -8, \frac{-2}{3} \right\} \)  
14. \( x = \left\{ \frac{3}{8}, \frac{7}{2} \right\} \)  
15. \( x = \left\{ \frac{1}{2} \right\} \)  
16. \( x = \left\{ \frac{3}{10} \right\} \)  
17. \([1, 7]\)  
18. \([-9, 7]\)  
19. \((-1, 4)\)  
20. \(\left\{ \frac{1}{2}, 1 \right\}\)  
21. \([-6, 2]\)  
22. \(\left[ \frac{-8}{3}, 4 \right]\)  
23. \(\phi\)  
24. \(\phi\)  
25. \(\phi\)  
26. \(\phi\)  
27. \(x = \{5\}\)  
28. \(x = \{3\}\)  
29. \((-\infty, -3] \cup [7, \infty)\)  
30. \((-\infty, -7] \cup [1, \infty)\)  
31. \((-\infty, -\frac{7}{3}) \cup (1, \infty)\)  
32. \((-\infty, \frac{3}{5}) \cup (1, \infty)\)  
33. \((-\infty, -\frac{9}{2}) \cup \left[ \frac{3}{2}, \infty \right]\)  
34. \((-\infty, -5] \cup \left[ \frac{5}{3}, \infty \right]\)  
35. \(x = (-\infty, 5) \cup (5, \infty)\)  
36. \(x = (-\infty, 3) \cup (3, \infty)\)  
37. \(\mathbb{R}\)  
38. \(\mathbb{R}\)  
39. \(\mathbb{R}\)  
40. \(\mathbb{R}\)  
41. a) \(\{-2, 1\}\), b) \((-\infty, -2] \cup [1, \infty)\), c) \([-2, 1]\)  
42. a) \(\{-4, 2\}\), b) \((-\infty, -4] \cup [2, \infty)\), c) \([-4, 2]\)  
43. a) \(\{-5.5, 3.5\}\), b) \((-\infty, -5.5) \cup (3.5, \infty)\), c) \((-5.5, 3.5)\)  
44. a) \(\phi\), b) \(\mathbb{R}\), c) \(\phi\)  
45. a) \(\{-6.5, 4.5\}\), b) \((-\infty, -6.5] \cup [4.5, \infty)\), c) \([-6.5, 4.5]\)  
46. a) \(\phi\), b) \(\mathbb{R}\), c) \(\phi\) 

Answers - Exercises 2.4  

Note: The symbol “+” is used if the boundary point is part of the solution and the symbol “-” is used when the boundary point is not part of the solution.

1. \(1, -8, 3\)  
2. \(-15, -7, 1\)  
3. \(1, 3, 5, 8\)  
4. \(2, -4, 1, 3\)  
5. \(3, 1, 7\)  
6. \(0, -5, 1\)  
7. \(4, 1, 4\)  
8. \(0, -7, -9\)  

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9. 0, -3, 2, -5, 11
10. 0, -6, 7, -8, 5
11. -5, -7, 2, 1, 3
12. -3, -4, 1, 3, 6
13. -1, -5, 1, 3, 8
14. -2, -2, 1, 8, 10
15. 2, 2, 1, 7, 8
16. -9, -5, -4, 1, -1
17. -2, 1, 2, 4, 9
18. -9, -5, -3, -1, 2

Answers - Exercises 2.5

1) 6x + 2  2) -2x - 8  3) 8x^2 - 2x - 15  4) \( \frac{2x - 3}{4x + 5}, x \neq -\frac{5}{4} \)
5) 8  6) 13  7) -4  8) 8
9) \( \frac{3}{17} \)  10) \(-\frac{1}{5}\)  11) 3x + 1  12) x + 7  13) 2x^2 - 2x - 12  14) \( \frac{2x + 4}{x - 3}; x \neq 3 \)
15) 7
16) -8  17) 4  18) -7  19) -3  20) \( \frac{1}{12} \)
21) 8x + 11  22) 8x + 1  23) 16x - 5
24) 4x + 9  25) 19  26) -15  27) -21  28) 9  29) 9x^2 - 6x - 2  30) 3x^2 - 10
31) x^4 - 6x^2 + 6  32) 9x - 4  33) 22  34) -10  35) -2  36) 5  37) 2
38) 3  39) 5  40) 3  41) 2x - 3 + h  42) 2x + 2 + h  43) 2x - 2 + h  44) 2x + 4 + h