Math 220 Big Ideas – Chapters 12 & 13

1. Three – Dimensional Coordinate System
   • Points in 3-D space, including notation and the distance between points
   • Planes parallel to coordinate planes and planes parallel to coordinate axes
   • Equation of Spheres
   • Cylindrical surfaces

2. Vectors
   a. Definition and length of vectors, vector addition, and scalar multiplication
   b. Geometric and algebraic representations
   c. The relationship between the vector representation \( \mathbf{a} = (a_1, a_2) \), the point \( P(a_1, a_2) \) and the position representation \( \overrightarrow{OP} \).
   d. Unit vectors, standard basis vectors,
   e. Determine whether vectors are equivalent, parallel , or orthogonal
   f. Vector multiplication
      i. Scalar multiplication
      ii. Dot product: algebraic and geometric formulas and applications
      iii. Cross product: algebraic and geometric formulas and applications
   g. Determine angles between vectors including direction angles
   h. Projections
      i. Properties of vectors
         i. Commutatively
         ii. Distributive

3. Lines in 3-space
   a. Requires point and parallel vector
   b. Vector equation
   c. Parametric equations
   d. Symmetric equations
   e. Skew lines

4. Planes in 3-space
   a. Requires point and normal vector and problem solving

5. Quadric surfaces
   a. Matching graphs with equations
   b. Use cross sections (or traces) to visualize graphs in 3D
   c. Solids of revolution

6. Vector valued functions
   a. Space curves, tangent lines and unit tangent vectors
   b. Derivatives & Integrals of vector valued functions

7. Arc Length and curvature
   a. Arc length of space curves
   b. Parameterize a curve with respect to arc length
   c. Curvature - definition
   d. Osculating circle and osculating plane
   e. Normal and binormal vectors

8. Velocity and Acceleration
   a. Definition of velocity and acceleration as vector functions
   b. Derive velocity from acceleration and position vectors
   c. Tangential and normal components of acceleration
1. Show the given equation is a sphere. Find its center and radius.
   \[2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1\]

2. Determine if these points are collinear. \(P(1, 2, 3), Q(5, -3, 8)\) and \(R(-7, 12, -7)\).

3. Determine if \(L_1\) and \(L_2\) are parallel, skew or intersecting. If they intersect, find the point of intersection. \(L_1: \frac{x-4}{2} = \frac{y+5}{4} = \frac{z-1}{-3}\) and \(L_2: \frac{x-2}{1} = \frac{y+1}{3} = \frac{z}{-2}\)

4. Find the point at which the given line intersects the given plane.
   \[x = 3 - t, y = 2 + t, z = 5t ; x - y + 2z = 9\]

5. Find the equation of the plane that passes through the point \((6, 0, -2)\) and contains the line \(x = 4 - 2t, y = 3 + 5t, z = 7 + 4t\).

6. Find the values of \(x\) such that the vectors \((6, -5, x)\) and \((-4, x, x)\) are orthogonal.

7. Find the distance between the given parallel planes.
   \[2x - 3y + z = 4, 4x - 6y + 2z = 3\]

8. Problems similar to #13 in section 12.4.

9. Identify the surface and sketch it. \(x^2 + 2z^2 - 6x - y + 10 = 0\)

10. Find symmetric equations of the line that is normal to the plane \(2x + 5y - 3z = 15\) at the point where the plane intersects the \(y\)-axis.

11. Find the parametric equations for the tangent line to the given curve at the given point.
    \[r(t) = (1 + 2t, 1 + t - t^2, 1 - t + t^2 - t^3), (1, 1, 1)\]

12. Find the length of the curve on the given interval.
    \[r(t) = (2t, 3 \sin t, 3 \cos t), 3 \leq t \leq 14\]

13. A point moves along a curve with \(\mathbf{a}(t) = (3, 6t, 4)\). Find its position function \(\mathbf{r}(t)\) if \(\mathbf{r}(1) = (2, -3, 5)\) and \(\mathbf{v}(1) = (1, -1, 1)\).

14. Evaluate this integral: \(\int_0^{t/4} (\cos 2t, \sin 2t, t \sin t) \, dt\)

15. Find the unit tangent vector \(\mathbf{T}(t)\), the unit normal vector \(\mathbf{N}(t)\) and the curvature \(\kappa(t)\) for the given curve. \(r(t) = (t, 3 \cos t, 3 \sin t)\)

16. Find the tangential and normal components of the acceleration vector for the given curve.
    \[r(t) = (t, t^2, 3t)\]

17. Given the vectors \(\mathbf{a} = \mathbf{i} + \mathbf{k}\) and \(\mathbf{b} = \mathbf{i} - \mathbf{j}\), find the scalar and vector projection of \(\mathbf{b}\) onto \(\mathbf{a}\) and find the angle between the two vectors.
Answers

1. Center (2, 0, −6) and radius is $\frac{9}{\sqrt{2}}$

2. Yes, collinear.

3. Intersecting, (−6, −25, 16)

4. (2, 3, 5)

5. $33x + 10y + 4z = 190$ or $-33(x - 6) - 10(y - 0) - 4(z + 2) = 0$

6. $x = 8$ or $x = -3$

7. $\frac{5}{2\sqrt{14}}$ or $\frac{5\sqrt{14}}{28}$

8. See book

9. Elliptic paraboloid with vertex (3, 1, 0) opening away from the $xz$-plane.

10. $\frac{x}{2} = \frac{y - 3}{5} = \frac{z}{-3}$

11. $x = 1 + 2t, y = 1 + t, z = 1 - t$

12. $11\sqrt{13}$

13. $r(t) = (\frac{3}{2}t^2 - 2t + \frac{5}{2}, t^3 - 4t, 2t^2 - 3t + 6)$

14. $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{\pi}{4\sqrt{2}}\right)$

15. $T(t) = \frac{1}{\sqrt{10}}(1, -3 \sin t, 3 \cos t), N(t) = (0, -\cos t, -\sin t), \kappa(t) = \frac{3}{10}$

16. $a_T = \frac{4t}{\sqrt{4t^2 + 10}}, a_N = \frac{2\sqrt{10}}{\sqrt{4t^2 + 10}}$

17. $\text{comp}_a b = \frac{1}{\sqrt{2}}, \text{proj}_a b = (\frac{1}{2}, 0, \frac{1}{2}), \theta = 60^\circ$