Intermediate Algebra Readiness Test

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Fractions

Simplifying fractions:

Example: Reduce 27/36:

\[
\frac{27}{36} = \frac{3 \times 3}{3 \times 12} = \frac{3}{4}
\]

(Note that you may need to find a common factor--in this case 9--in both the top and bottom in order to reduce.)

1. 1 / 3: Reduce:
   1. \( \frac{11}{32} \) = \( \frac{3}{6} \)
   2. \( \frac{26}{65} \) = \( \frac{3}{9} \)

Equivalent fractions:

Example: 3/4 is equivalent to how many eights?

\[
\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}
\]

4. To 5: Complete:
   4. \( \frac{3}{8} = \frac{72}{?} \)
   5. \( \frac{3}{2} = \frac{20}{?} \)

How to get the lowest common denominator (LCD) by finding the least common multiple (LCM) of all denominators:

Example: 5/6 and 8/15

First find LCD of 6 and 15:

- \( 6 = 2 \times 3 \)
- \( 15 = 3 \times 5 \)

LCM = 3 \times 5 \times 2 = 30

+ \( \frac{5}{6} = \frac{5 \times 5}{30} = \frac{25}{30} \)
- \( \frac{8}{15} = \frac{8 \times 2}{30} = \frac{16}{30} \)

6. To 7: Find equivalent fractions with the LCD:
   6. \( \frac{5}{6} \) and \( \frac{8}{9} \) | \( \frac{7}{8} \) and \( \frac{12}{16} \)

8. Which is larger, 5/7 or 3/11? (Hint: Find LCD fractions)

Adding and subtracting fractions:

Example: \( \frac{7}{10} - \frac{3}{10} = \frac{7 - 3}{10} = \frac{4}{10} = \frac{2}{5} \)

9. To 11: Find the sum or difference (reduce if possible):
   9. \( \frac{1}{2} + \frac{3}{4} = \frac{7}{4} \)
   10. \( \frac{5}{6} + \frac{1}{9} = \frac{11}{18} \)

If denominators are different, find equivalent fractions with common denominators, then proceed as before:

Example:

\[
\frac{3}{5} + \frac{4}{12} = \frac{3 \times 12}{5 \times 12} + \frac{4 \times 5}{12 \times 5} = \frac{36}{60} + \frac{20}{60} = \frac{56}{60} = \frac{14}{15}
\]

12. \( \frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{6}{4} = \frac{3}{2} = \frac{3}{2} \)

Multiplying fractions: multiply the tops, multiply the bottoms, reduce if possible:

Example:

\[
\frac{1}{4} \times \frac{2}{3} = \frac{1 \times 2}{4 \times 3} = \frac{2}{12} = \frac{1}{6}
\]

14. \( \frac{1}{3} \times \frac{2}{5} = \frac{2}{15} \)

15. \( \frac{1}{5} \times \frac{3}{7} = \frac{3}{35} \)

Dividing fractions: a nice way to do this is to make a compound fraction and then multiply the top and bottom (of the big fraction) by the LCD of both:

Example:

\[
\frac{1}{2} \div \frac{3}{4} = \frac{4}{3} \times \frac{1}{2} = \frac{4}{6} = \frac{2}{3}
\]

Example:

\[
\frac{2}{3} \div \frac{5}{6} = \frac{3}{5} \times \frac{6}{2} = \frac{18}{10} = \frac{9}{5}
\]

18. \( \frac{1}{2} \div \frac{3}{4} = \frac{4}{3} \times \frac{1}{2} = \frac{2}{3} \)

19. \( \frac{3}{5} \div \frac{4}{6} = \frac{3}{5} \times \frac{6}{4} = \frac{9}{20} \)

20. \( \frac{1}{2} \div \frac{2}{3} = \frac{2}{3} \times \frac{3}{2} = \frac{1}{2} \)

B. Decimals

Meaning of place: in 124,529,

each digit position has a value ten times the place to its right. The part to the left of the point is the whole number part. Right of the point, the places have values: tenths, hundredths, etc., so 124,529 = (1 \times 100,000) + (2 \times 10,000) + (4 \times 1,000) + (5 \times 100) + (2 \times 10) + (9 \times 1)

23. Which is larger: .59 or .77?

To add or subtract decimals, line up the points.

Example:

\[
1.23 \div -0.1 = 1.13
\]

Example:

\[
4.23 + 4.3 = 8.5
\]

60. \( 6.02 - 2.14 = 3.88 \)

24. \( 5.6 \times 7.8 = 43.88 \)

25. \( 1.36 = 5.63 \)

26. \( 4 \div 3.01 = 4.01 \)

27. \( 3.5 \times 9.16 = 32.31 \)

Multiplying decimals:

Example:

\[
-0.03 \times 0.03 = 0.0009
\]

28. \( 0.3 \times 10 = 3.0 \)

29. \( 0.01 \times 2 = 0.02 \)

Dividing decimals: change the problem to an equivalent whole number problem by multiplying both by the same power of ten.

Example:

\[
-0.03 \div 0.001 = 30 \times \frac{1}{10} = 300
\]

Example:

\[
0.001 \div 0.07 = 100 \times \frac{1}{100} = 10
\]

32. \( 0.03 \div 100 = 300 \times \frac{1}{10} = 3
\]

33. \( 0.05 \div 2 = 200 \times \frac{1}{100} = 2
\]

C. Positive Integer exponents

and square roots of perfect squares

Meaning of exponents (powers):

Example:

\[
j^2 = 3 \times 3 = 9
\]

Example:

\[
x^3 = 4 \times 4 \times 4 = 64
\]

35. \( j^2 = 16 \times 100 = 1,000 \)

36. \( (3)^2 = 3 \times 3 = 9 \)

37. \( (-3)^2 = 3 \times 3 = 9 \)

38. \( -3^2 = 3 \times -3 = -9 \)

39. \( (-2)^3 = -2 \times -2 \times -2 = -8 \)

\( \sqrt{a} \) is a non-negative real number if \( a \geq 0 \)

\( \sqrt{b} = b \) means \( b^2 = a \), where \( b \geq 0 \). Thus \( \sqrt{49} = 7 \), because \( 7^2 = 49 \).

Also, \( -\sqrt{49} = -7 \)

45. \( \sqrt{144} = 12 \)

46. \( -\sqrt{144} = 12 \)

47. \( -\sqrt{144} = 12 \)

49. \( \sqrt{100} = 10 \)

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D. Fraction-decimal conversion

Fraction to decimal: divide the top by the bottom.

Example: \( \frac{1}{4} = 1 \div 4 = 0.25 \)
Example: \( \frac{5}{8} = 5 \div 8 = 0.625 \)
Example: \( \frac{3}{8} = 3 \div 8 = 0.375 \)

52 to 55: Write each as a decimal. If the decimal repeats, show the repeating block of digits.

52. \( \frac{1}{6} = 0.16666... = 0.\overline{1} \)
53. \( \frac{3}{4} = 3 \div 4 = 0.75 \)

Non-repeating decimals to fractions: read the number as a fraction, write it as a fraction, reduce if possible.

Example: \( 0.04 = \frac{4}{100} = \frac{1}{25} \)
Example: \( 3.76 = \frac{376}{100} = \frac{94}{25} \)

56 to 58: Write as a fraction.

56. \( 0.01 = \frac{1}{100} \)
57. \( 4.9 = \frac{49}{10} \)

H. Percent

Meaning of percent: translate "percent" as "per hundred":

Example: 8% means 8 hundredths or \( \frac{8}{100} = \frac{2}{25} \)

To change a decimal to percent, multiply by 100 and move the point 2 places right and write the percent symbol (%).

Example: \( 0.075 = 7.5\% \)
Example: \( 0.4 = 0.25 = 25\% \)

59 to 60: Write as a percent.

59. \( 0.01 = 1\% \)
60. \( 0.4 = 40\% \)

To change a percent to a decimal, move the point 2 places left and drop the % symbol.

Example: \( 8.78\% = 0.0878 \)
Example: \( 67\% = 0.67 \)

61 to 62: Write as a decimal.

61. \( 10\% = 0.1 \)
62. \( 40\% = 0.4 \)

To solve a percent problem which can be written in this form: \( a \% \) of \( b \) is \( c \)

First identify \( a \), \( b \), \( c \):

63. \( 6\% \) of \( 100 \) is \( 1.2 \)
64. \( 600 \) is \( 15\% \) of \( 4000 \)
65. \( 3 \) out of \( 12 \) is \( 25\% \)

Given \( a \) and \( b \), change \( a \% \) to decimal form and multiply (since "of" can be translated "multiply").

Example: What is \( 9.4\% \) of \( \$5000 \)?
\[ (\text{as } b \text{ is } 100, 9.4\% \text{ of } 5000 \text{ is } \frac{9.4}{100} \times 5000 = \$470 \) \]

Example: 56 problems right out of 80 is what percent?
\[ \frac{56}{80} = 0.7 = 70\% \] \( \text{(answer)} \)

Example: 5610 people vote in an election, which is 60% of the registered voters. How many are registered?
\[ \frac{60}{100} \text{ of } x = 5610 \]
\[ x = \frac{5610}{0.60} = 9350 \] \( \text{(answer)} \)

66. \( 4\% \) of 9 is what?

67. What percent of 70 is 58?

68. 15% of what is 60?

P. Estimation and approximation

Rounding to one significant digit:

Example: 3.67 rounds to 4
Example: 0.049 rounds to 0
Example: 850 rounds to either 800 or 900

69 to 71: Round to one significant digit:

69. 45.01
70. 1.09

To estimate an answer, it is often sufficient to round each given number to one significant digit, then compute.

Example: \( 0.028 \times 0.0005 = 0.000014 \)
Round and compute:
\[ 0.01 \times 0.005 = 0.00005 \]
\[ 0.000015 \text{ is the estimate} \]

72 to 75: Select the best approximation of the answer:

72. 1.23\( \times 10^7 \) x 367,002,960 = \( 4.40, 400, 4000, 40000 \)
73. \( 0.01221098 \div 0.190498238 = (0.02, 0.2, 0.5, 20, 50) \)
74. 101.7293507 + 3.121392653 = \( 2, 4, 98, 105, 400 \)
75. \( 1.362859293 \times \)
\[ 12, 64, 640, 5000, 12000 \]
Intermediate Algebra Readiness Test

Topic 2: Polynomials

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Grouping to simplify polynomials

The distributive property says \( a(b + c) = ab + ac \)

Example:

\[ 3(x - y) = 3x - 3y \]
(a = 3, b = x, c = -y)

Example:

\[ 4x + 7x = (4 + 7)x = 11x \]
(a = x, b = 4, c = 7)

Example:

\[ 4a + 6x - 2 = 2(2a + 3x - 1) \]

1 to 3: Rewrite, using the distributive property:

1. \[ 6(x - 3) = \]

2. \[ 4x - x = \]

3. \[ -5(a - 1) = \]

Commutative and associative properties are also used in regrouping:

Example:

\[ 3x + 7 - x = 3x + x + 7 = 2x + 7 \]

Example:

\[ 5 - x + 5 = 5 + 5 - x = 10 - x \]

Example:

\[ 3x + 2y - 2x + 3y = 3x - 2x + 2y + 3y = x + 5y \]

4 to 9: Simplify:

4. \[ x + x = \]

5. \[ a + b - a + b = \]

6. \[ 9x - y + 3y - 8x = \]

7. \[ 4x + 1 + x - 2 = \]

8. \[ 160 - x - 90 = \]

9. \[ x - 2y + y - 2x = \]

B. Evaluation by substitution

Example:

If \( x = 3 \), then
\[ 7 - 4x = 7 - 4(3) = 7 - 12 = -5 \]

Example:

If \( a = -7 \) and
\( b = -1 \), then \( a^2b = (-7)^2(-1) = 49(-1) = -49 \)

Example:

If \( x = -2 \), then
\[ 3x^2 - x - 5 = 3(-2)^2 - (-2) - 5 = 12 + 2 - 5 = 9 \]

10 to 19: Given \( x = -1 \),
\[ y = 3; z = -3. \]
Find the value:

10. \( 2x = \]

11. \( -a = \]

12. \( xx = \]

13. \( y + z = \]

14. \( y^2 + z^2 = \]

15. \( 2x + 4y = \]

16. \( 2x^2 - x - 1 = \]

C. Adding, subtracting polynomials

Combine like terms:

Example:

\[ (3x^2 + x + 1) - (x - 1) = 3x^2 + x + 1 - x + 1 = 3x^2 + 2 \]

Example:

\[ (x - 13) + (x^2 + 2x - 3) = x + x^2 + 2x - 3 = x^2 + 3x - 4 \]

Example:

\[ (x^2 + x - 1) - (6x^2 + 2x + 1) = x^2 + x - 1 - 6x^2 - 2x - 1 = -5x^2 - x - 2 \]

D. Monomial times polynomial

Use the distributive property:

Example:

\[ 3(x - 4) = \]

\[ 3x + (-12) = 3x - 12 \]

Example:

\[ (2x + 3)a = \]

\[ 2ax + 3a \]

Example:

\[ -4x(x^2 - 1) = -4x^3 + 4x \]

26. \( -(x - 7) = \]

27. \( -2(x + a) = \]

28. \( x(x + 5) = \]

29. \( (3x - 1)7 = \]

30. \( a(2x - 3) = \]

31. \( (x^2 - 1)(-1) = \]

32. \( 8(a^2 + 2a - 7) = \]

E. Multiplying polynomials; use the distributive property:

\( a(b + c) = ab + ac \)

Example:

\[ (2x + 1)(x - 4) = \]

\[ (2x + 1)x + (2x + 1)(-4) = 2x^2 + x - 8x - 4 = 2x^2 - 7x - 4 \]

F. First times First:

\( (2x)(x) = 2x^2 \)

0: multiply 'Outers':

\( (2x)(-4) = -8x \)

I: multiply 'Inners':

\( (1)(x) = x \)

L: Last times Last:

\( (1)(-4) = -4 \)

Add, get \( 2x^2 - 7x - 4 \)
examples:

\[(x + 2)(x + 3) = \]
\[x^2 + 5x + 6\]
\[(2x - 1)(x + 2) = \]
\[2x^2 + 3x - 2\]
\[(x - 5)(x + 5) = x^2 - 25\]
\[-4(x - 3) = -4x + 12\]
\[(3x - 4)^2 = \]
\[9x^2 - 24x + 16\]
\[(x + 3)(x - 5) = \]
\[ax^2 + 5x + 3a - 15\]

33 to 44: Multiply:
33. \((x + 3)^2 = \)
34. \((x - 3)^2 = \)
35. \((x + 3)(x - 3) = \)
36. \((2x + 3)(2x - 3) = \)
37. \((x - 1)(x + 2) = \)
38. \(-6x(3 - x) = \)
39. \((x - \frac{3}{2})^2 = \)
40. \((x - 1)(x + 3) = \)
41. \((x^2 - 1)(x^2 + 3) = \)

0. Factoring

Monomial factors:
ab + ac = a(b + c)

examples:
\[x^2 - x(x - 1) = \]
\[4x^2 + 6xy + 2xy(2x + 3) = \]

difference of two squares:
\[a^2 - b^2 = (a + b)(a - b)\]

examples:
\[9x^2 - 4 = \]
\[(3x + 2)(3x - 2)\]

Trinomial square:
\[a^2 + 2ab + b^2 = (a + b)^2\]
\[a^2 - 2ab + b^2 = (a - b)^2\]

examples:
\[x^2 - 6x + 9 = (x - 3)^2\]

P. Special Products

These product patterns (examples of FOIL) should be remembered and recognized:

I. \((a + b)(a - b) = \]
\[a^2 - b^2\]

II. \((a + b)^2 = \]
\[a^2 + 2ab + b^2\]

III. \((a - b)^2 = \]
\[a^2 - 2ab + b^2\]

example 1:
\[(3x - 1)^2 = 9x^2 - 6x + 1\]
example 2:
\[(x + 5)^2 = x^2 + 10x + 25\]
example 3:
\[(x + 8)(x - 8) = x^2 - 64\]

42 to 44: Match each pattern with its example:
42. I: 43. II: 44. III:
INTERMEDIATE ALGEBRA READINESS TEST
Topic 3: Linear equations and inequalities

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Solving one linear equation in one variable: add or subtract the same thing on each side of the equation, or multiply or divide each side by the same thing, with the goal of getting the variable alone on one side. If there are one or more fractions, it may be desirable to eliminate them by multiplying both sides by the common denominator. If the equation is a proportion, you may wish to cross-multiply.

1. \(2x = 9\)
   \(x = \frac{9}{2}\)

2. \(j = \frac{6x}{5}\)
   \(j = \frac{6x}{5}\)

3. \(3x + 7 = 6\)
   \(3x = -1\)
   \(x = -\frac{1}{3}\)

4. \(\frac{x}{7} = 5\)
   \(x = 35\)

5. \(5 - x = 9\)
   \(x = -4\)

6. \(x = \frac{2x}{5} + 1\)
   \(x = 5\)

To solve a linear equation for one variable in terms of the other(s), do the same as above:

Example: Solve for \(F\): \(C = \frac{5}{9}(F - 32)\)

Multiply by \(\frac{9}{5}\):
\(C\frac{9}{5} = F - 32\)

Add 32:
\(C\frac{9}{5} + 32 = F\)

Thus, \(F = C\frac{9}{5} + 32\).

Example: Solve for \(b\): \(a + b = 90\)

Subtract \(a\): \(b = 90 - a\)

Example: Solve for \(x\): \(ax + b = c\)

Subtract \(b\): \(ax = c - b\)

Divide by \(a\): \(x = \frac{c - b}{a}\)

B. Solution of a one-variable equation reducible to a linear equation:

Some equations which don’t appear linear can be solved by using a related linear equation:

Example: \(\frac{x + 1}{3x} = -1\)

Multiply by \(3x\): \(x + 1 = -3x\)

Solve:
\(4x = -1\)
\(x = -\frac{1}{4}\)

(Use sure to check answer in the original equation.)

Example: \(\frac{2x + 3}{x + 1} = 5\)

Think of \(x + 1\) as \(\frac{1}{5}\) and

cross-multiply:
\(5x + 5 = 3x + 3\)

\(2x = -2\)

\(x = -1\)

But \(-1\) doesn’t make the original equation true (doesn’t check), so there is no solution.

20 to 25: Solve and check:

20. \(\frac{x - 1}{x + 1} = 6\)

21. \(\frac{2x + 3}{x + 1} = \frac{5}{2}\)

22. \(\frac{3x - 2}{2x + 1} = 4\)

23. \(\frac{x + 3}{2x} = 2\)

24. \(\frac{1}{3} = \frac{x}{x + 8}\)

25. \(\frac{x - 2}{4 - 2x} = 3\)

Example: \(|3 - x| = 2\)

Since the absolute value of both 2 and -2 is 2, \(3 - x\) can be either 2 or -2. Write these two equations and solve each:

\(3 - x = 2\) or \(3 - x = -2\)

\(-x = -1\) or \(-x = 5\)

\(x = 1\) or \(x = 5\)

26 to 30: Solve:

26. \(|x| = 3\)

27. \(|x| = -1\)

28. \(|x - 1| = 3\)

29. \(|2 - 3x| = 0\)

30. \(|x + 2| = 1\)
G. Solution of linear inequalities

Rules for inequalities:

<table>
<thead>
<tr>
<th>If a &gt; b, then:</th>
<th>If a &lt; b, then:</th>
</tr>
</thead>
<tbody>
<tr>
<td>a + c &gt; b + c</td>
<td>a + c &lt; b + c</td>
</tr>
<tr>
<td>a - c &lt; b - c</td>
<td>a - c &gt; b - c</td>
</tr>
<tr>
<td>ac &gt; bc (if c&gt;0)</td>
<td>ac &lt; bc (if c&gt;0)</td>
</tr>
<tr>
<td>ac &lt; bc (if c&lt;0)</td>
<td>ac &gt; bc (if c&lt;0)</td>
</tr>
<tr>
<td>a/b &gt; c (if c&gt;0)</td>
<td>a/b &lt; c (if c&gt;0)</td>
</tr>
<tr>
<td>a/b &lt; c (if c&lt;0)</td>
<td>a/b &gt; c (if c&lt;0)</td>
</tr>
</tbody>
</table>

Example: One variable graph: solve and graph on number line: \( 1 - 2x \leq 7 \)
(This is an abbreviation for \( \{x: 1 - 2x \leq 7\}\))
Subtract 1, get \(-2x \leq 6\)
Divide by -2, \(x \geq -3\)
Graph: \(-4 -3 -2 -1 0 1 2 3\)

31 to 38: Solve and graph on number line:

31. \(x - 3 \geq 4\)  
32. \(-4x < 2\)  
33. \(2x + 1 \leq 6\)  
34. \(3 \leq x = 3\)  
35. \(4 = 2x < 6\)  
36. \(5 - x > x - 3\)  
37. \(x > 1 + 4\)  
38. \(6x + 5 \geq 4x - 3\)

D. Solving a pair of linear equations in two variables: the solution consists of an ordered pair, an infinite number of ordered pairs, or no solution.

39 to 46: Solve for the common solution(s) by substitution or linear combinations:

<table>
<thead>
<tr>
<th>39. (x + 2y = 7)</th>
<th>43. (2x - 3y = 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3x - y = 26)</td>
<td>(3x + 5y = 1)</td>
</tr>
<tr>
<td>40. (x + y = 5)</td>
<td>44. (4x - 1 = y)</td>
</tr>
<tr>
<td>(x - y = -3)</td>
<td>(4x + y = 1)</td>
</tr>
<tr>
<td>41. (2x - y = -9)</td>
<td>45. (x + y = 3)</td>
</tr>
<tr>
<td>(x = 8)</td>
<td>(x + y = 1)</td>
</tr>
<tr>
<td>42. (2x - y = 1)</td>
<td>46. (2x - y = 3)</td>
</tr>
<tr>
<td>(y = x - 5)</td>
<td>(6x - 9 = 3y)</td>
</tr>
</tbody>
</table>

Examples:

1. \(9/2\)
2. \(5/2\)
3. \(-1/3\)
4. \(15/4\)
5. \(-4\)
6. \(5/7\)
7. \(2\)
8. \(10\)
9. \(6/5\)
10. \(3\)
11. \(-1/3\)
12. \(160 - 4\)
13. \(90 - a\)
14. \((r - 2h)/2\)
15. \((y + 2)/3\)
16. \(4 - y\)
17. \((3y - 1)/2\)
18. \(-4/h\)
19. \(x/b\)
20. \(13\)
21. \(-5/4\)
22. \(-6/5\)
23. \(1\)
24. \(4\)
25. \(b/2\)
26. \(3\)
27. \(5\)
28. \(-2\)
29. \(3/3\)
30. \(3\)
31. \(x = 7\)
32. \(x = 1/2\)
33. \(x = 5/2\)
34. \(x = 6\)
35. \(x = -1\)
36. \(x = 0\)
37. \(x = 5\)
38. \(x = -4\)
39. \(a = 1\)
40. \((1, b)\)
41. \((0, 25)\)
42. \((-4, 6)\)
43. \((-6/5, -13/19)\)
44. \((3/4, 0)\)
45. \(b/2\)
46. Any ordered pair of the form \((a, 2x - 3)\) where \(a\) is any number. One example: \((1, 5)\). Infinitely many solutions.

47. No solution
INTERMEDIATE ALGEBRA READINESS TEST

Topic 4: Quadratic equations

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. \( ax^2 + bx + c = 0 \); a quadratic equation can always be written so it looks like
\[ ax^2 + bx + c = 0 \]
where \( a \), \( b \), and \( c \) are real numbers and \( a \) is not zero.

**Example:** \( 5 - x = 3x^2 \)

Add \( x \):
\[ 5 = 3x^2 + x \]
Subtract 5:
\[ 0 = 3x^2 + x - 5 \]
or
\[ 3x^2 + x - 5 = 0 \]
So \( a = 3 \), \( b = 1 \), \( c = -5 \)

**Example:** \( x^2 = 3 \)

Rewrite:
\[ x^2 = 3 \]
(Think of \( x^2 + 0x - 3 = 0 \))
So \( a = 1 \), \( b = 0 \), \( c = -3 \)

1 to 4: Write each of the following in the form \( ax^2 + bx + c = 0 \) and identify \( a \), \( b \), \( c \):

1. \( 3x + x^2 - 4 = 0 \)
2. \( 5 - x^2 = 0 \)
3. \( x^2 = 3x - 1 \)
4. \( x = 3x^2 \)
5. \( 8x^2 = 1 \)

B. Factoring

**Monomial factors:**
\[ ab + ac = a(b + c) \]

**Examples:**
\[ x^2 - x = x(x - 1) \]
\[ 4x^2y + 6xy = 2xy(2x + 3) \]

**Difference of two squares:**
\[ a^2 - b^2 = (a + b)(a - b) \]

**Example:**
\[ 9x^2 - 4 = (3x + 2)(3x - 2) \]

**Trinomial square:**
\[ a^2 + 2ab + b^2 = (a + b)^2 \]
\[ a^2 - 2ab + b^2 = (a - b)^2 \]

**Example:**
\[ x^2 - 6x + 9 = (x - 3)^2 \]

**Trinomial:**

**Examples:**
\[ x^2 - x - 2 = (x - 2)(x + 1) \]
\[ 6x^2 - 7x - 3 = (3x + 1)(2x - 3) \]

6 to 20: Factor:

6. \( a^2 + ab \)
7. \( a^3 - a^2b + ab^2 \)
8. \( 8x^2 - 2 \)
9. \( x^2 - 10x + 25 \)
10. \( -4xy + 10x^2 \)
11. \( 2x^2 - 3x - 5 \)
12. \( x^2 - x - 6 \)
13. \( x^2 - y^2 \)
14. \( x^2 + 3x - 10 \)
15. \( 2x^2 - x \)
16. \( 2x^3 + 8x^2 + 8x \)
17. \( 9x^2 + 12x + 4 \)
18. \( 6x^3y^2 - 9x^2y \)
19. \( 1 - x - 2x^2 \)
20. \( 3x^2 - 10x + 3 \)

C. Solving factored quadratic equations; the following statement is the central principle:

\[ \text{If } ab = 0, \text{ then } a = 0 \text{ or } b = 0 \]

First, identify \( a \) and \( b \) in \( ab = 0 \):

**Example:** \((3 - x)(x + 2) = 0\)

Compare this with \( ab = 0 \):
\[ a = (3 - x) \]
\[ b = (x + 2) \]

21 to 24: Identify \( a \) and \( b \) in each of the following:

21. \( 3x(2x - 5) = 0 \)
22. \( (x - 3)x = 0 \)
23. \( (2x - 1)(x - 5) = 0 \)
24. \( 0 = (x - 1)(x + 1) \)

Then, because \( ab = 0 \) means \( a = 0 \) or \( b = 0 \), we can use the factors to make two linear equations to solve.
example: if $2x(3x - 4) = 0$
then $(2x) = 0$ or $(3x - 4) = 0$
and so $x = 0$ or $3x = 4$
$x = \frac{4}{3}$

Thus, there are two solutions:
$0$ and $\frac{4}{3}$

example: if $(3 - x)(x + 2) = 0$
then $(3 - x) = 0$ or $(x + 2) = 0$
and thus $x = 3$ or $x = -2$

example: if $(2x + 7)^2 = 0$
then $2x + 7 = 0$
$2x = -7$
$x = -\frac{7}{2}$ (one solution)

Note: there must be a zero on one side of the equation to solve by the factoring method.

25 to 31: Solve:
25. $(x + 1)(x - 1) = 0$
26. $4x(x + 4) = 0$
27. $0 = (2x - 5)x$
28. $0 = (2x + 3)(x - 1)$
29. $(x - 6)(x - 6) = 0$
30. $(2x - 3)^2 = 0$
31. $x(x + 2)(x - 3) = 0$

D. Solving quadratic equations by factoring; arrange the equation so zero is on one side (in the form $ax^2 + bx + c = 0$), factor, set each factor equal to zero, and solve the resulting linear equations.

example: solve $6x^2 = 3x$
Rewrite: $6x^2 - 3x = 0$
Factor: $3(2x - 1) = 0$
So $3x = 0$ or $(2x - 1) = 0$
Thus $x = 0$ or $x = \frac{1}{2}$

example: $0 = x^2 - x - 12$
$0 = (x - 4)(x + 3)$
$x - 4 = 0$ or $x + 3 = 0$
x = 4 or x = -3

32 to 43: Solve by factoring:
32. $x(x - 3) = 0$
33. $x^2 - 2x = 0$
34. $2x^2 = x$
35. $3(x + 4) = 0$
36. $x^2 = 2 - x$
37. $x^2 + x = 6$
38. $0 = (x + 2)(x - 3)$
39. $(2x + 1)(x - 2) = 0$
40. $6x^2 = x + 2$
41. $9 + x^2 = 6x$
42. $1 - x = 2x^2$
43. $x^2 - x - 6 = 0$

Another problem form: if a problem is stated in this form: 'One of the solutions of $ax^2 + bx + c = 0$ is $d$', solve the equation as above, then verify the statement.

example: Problem: One of the solutions of $10x^2 - 5x = 0$ is
A. $-\frac{2}{5}$
B. $\frac{1}{2}$
C. $\frac{1}{10}$
D. $2$
E. $5$

Solve $10x^2 - 5x = 0$ by factoring:
a. $5x(2x - 1) = 0$
so $5x = 0$ or $2x - 1 = 0$
thus $x = 0$ or $x = \frac{1}{2}$

Since $x = \frac{1}{2}$ is one solution,
answer C is correct.

44. One of the solutions of $(x - 1)(3x + 2) = 0$ is
A. $-\frac{1}{3}$
B. $-\frac{2}{3}$
C. 0
D. 2
E. 5

45. One solution of $x^2 - x - 2 = 0$ is
A. $-2$
B. $-1$
C. $-\frac{1}{2}$
D. $\frac{1}{2}$
E. 1

Appendix:

Answers:

<table>
<thead>
<tr>
<th>$x^2 + 3x - 4 = 0$</th>
<th>$x^2 + 3x - 1 = 0$</th>
<th>$x^2 - 3x - 4 = 0$</th>
<th>$x^2 - 3x - 1 = 0$</th>
<th>$x^2 - 4x + 4 = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1 = 1$</td>
<td>$x_2 = -1, 0, 5$</td>
<td>$x_2 = 1$</td>
<td>$x_2 = 1$</td>
<td>$x_2 = 0, 3$</td>
</tr>
<tr>
<td>$x_2 = -3, 4$</td>
<td>$x_2 = 1$</td>
<td>$x_2 = 1$</td>
<td>$x_2 = 1$</td>
<td>$x_2 = 1$</td>
</tr>
<tr>
<td>$x_2 = 2$</td>
<td>$x_2 = 1$</td>
<td>$x_2 = 1$</td>
<td>$x_2 = 1$</td>
<td>$x_2 = 1$</td>
</tr>
</tbody>
</table>

(Rotate on 1 to 5: All signs could be opposite)

21. $2a^2 - 2ab + b^2 = 0$
22. $a^2 - 2ab + b^2 = 0$
23. $2x^2 - 2x + 1 = 0$
24. $x^2 - 1 = 0$
25. $2x^2 - x + 3 = 0$
26. $x^2 - 2x + 1 = 0$
27. $x^2 + 2x + 1 = 0$
28. $x^2 + 2x + 2 = 0$
29. $x^2 + 2x + 3 = 0$
30. $x^2 + 2x + 4 = 0$
**Intermediate Algebra Readiness Test**

**Topic 5: Graphing**

**Directions:** Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Graphing a point on the number line

1 to 7: Select the letter of the point on the number line with coordinate:

1. 0
2. -2
3. -1
4. 1
5. 1.5
6. 2.75
7. -3

8 to 10: Which letter best locates the given number:

8. -5
9. 3
10. 2

11 to 13: Solve each equation and graph the solution on the number line:

**Example:**

\[ x + 3 = 1 \]

\[ x = -2 \]

11. \[ 2x - 6 = 0 \]
12. \[ x = 3x - 5 \]

B. Graphing a linear inequality (in one variable) on the number line

**Rules for inequalities:**

If \( a > b \), then: \( a + c > b + c \)

If \( a < b \), then: \( a + c < b + c \)

If \( a > b \) and \( c > 0 \), then: \( ac > bc \)

If \( a < b \) and \( c < 0 \), then: \( ac < bc \)

14 to 20: Solve and graph on number line:

14. \[ x - 3 > 4 \]
15. \[ 4x < 2 \]
16. \[ 2x + 1 < 6 \]
17. \[ 3 < x - 3 \]
18. \[ 4 - 2x < 6 \]
19. \[ 5 = x - 3 \]
20. \[ x > 1 + 4 \]

**Example:**

\[ x > -3 \text{ and } x < 1 \]

The two numbers \(-3\) and \(1\) split the number line into three parts: \(x < -3\), \(-3 < x < 1\), and \(x > 1\). Check each part to see if both \(x > -3\) and \(x < 1\) are true:

<table>
<thead>
<tr>
<th>part</th>
<th>(x &lt; -3)</th>
<th>(x &lt; 1)</th>
<th>both true?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>2</td>
<td>yes</td>
<td>yes (solution)</td>
<td>yes</td>
</tr>
<tr>
<td>3</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

Thus the solution is \(-3 < x < 1\) and the graph is:

11. \[ x \leq -3 \text{ or } x < 1 \]

12. \[ x = 3x - 5 \]

('or' means 'and/or')

<table>
<thead>
<tr>
<th>part</th>
<th>(x &lt; -3)</th>
<th>(x &lt; 1)</th>
<th>at least one true?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>yes</td>
<td>yes (solution)</td>
<td>yes</td>
</tr>
<tr>
<td>2</td>
<td>yes</td>
<td>yes (solution)</td>
<td>yes</td>
</tr>
<tr>
<td>3</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

So \(x < -3\) or \(-3 < x < 1\); these cases are both covered if \(x < 1\). Thus the solution is \(x < 1\) and the graph is:

21 to 23: Solve and graph:

21. \[ x < 1 \text{ or } x > 3 \]
22. \[ x > 0 \text{ and } x > 2 \]
23. \[ x > 1 \text{ and } x < 4 \]

C. Graphing a point in the coordinate plane

If two number lines intersect at right angles so that:

1) one is horizontal with positive to the right and negative to the left,
2) the other is vertical with positive up and negative down, and
3) the zero points coincide, then they form a coordinate plane, and

1) the horizontal number line is called the \(x\)-axis,
2) the vertical line is the \(y\)-axis,
3) the common zero point is the origin,
4) there are four quadrants, numbered as shown:

To locate a point on the plane, an ordered pair of numbers is used, written in the form \((x, y)\). The \(x\)-coordinate is always given first.
24 to 27: Identify $x$ and $y$ in each ordered pair:

24. $(3, 0)$
25. $(-2, 5)$
26. $(5, -2)$
27. $(0, 3)$

To plot a point, start at the origin and make the two moves, first in the $x$-direction (horizontal) and then in the $y$-direction (vertical) indicated by the ordered pair.

Example: $(-3, 4)$

Start at the origin, move left 3 (since $x = -3$), then (from there), up 4 (since $y = 4$). Put a dot there to indicate the point $(-3, 4)$.

28. Join the following points in the given order: $(-3, -2)$, $(1, -4)$, $(3, 0)$, $(2, 2)$, $(3, 0)$, $(-1, -2)$, $(-1, 2)$, $(1, 4)$.

29. Two of the lines you draw cross each other. What are the coordinates of this crossing point?

30. In what quadrant does the point $(a, b)$ lie, if $a > 0$ and $b < 0$?

31 to 34: For each given point, which of its coordinates, $x$ or $y$, is larger?

35 to 41: Given each line on the number plane and find its slope (refer to section B below if necessary):

35. $y = 3x$  
36. $x - y = 3$  
37. $x = 1 - y$  
38. $y = 1$

39. $x = -2$  
40. $y = -2x$  
41. $y = \frac{1}{2}x + 1$

E. Slope of a line through two points

42 to 47: Find the value of each of the following:

42. \( \frac{1}{2} = \frac{0 - 2}{1 - 0} \)
43. \( \frac{5 - 2}{1 - 0} = \frac{3}{1} \)
44. \( \frac{-6 - (-1)}{-2 - 0} = \frac{-5}{-2} \)

The line joining the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ has slope $\frac{y_2 - y_1}{x_2 - x_1}$.

Example: A $(3, -1)$, B $(-2, 4)$

Slope of AB $= \frac{4 - (-1)}{-2 - 3} = \frac{5}{-5} = -1$

48 to 54: Find the slope of the line joining the given points:

48. $(-3, 1)$ and $(-1, -4)$
49. $(0, 2)$ and $(-3, -5)$
50. $(3, -1)$ and $(5, -1)$

51. $x = -2$
52. $y = -2$
53. $x = 2$
54. $y = 2$
INTERMEDIATE ALGEBRA READINESS TEST
Topic 6: Rational expressions

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Simplifying fractional expressions

example: \( \frac{27}{36} = \frac{9 \cdot 3}{9 \cdot 4} = \frac{9}{4} = 1 \cdot \frac{3}{4} \) (note that you must be able to find a common factor—in this case 9—in both the top and bottom in order to reduce the fraction.)

example: \( \frac{3a}{12ab} = \frac{3a \cdot 1}{3a \cdot 4b} = \frac{3a}{4b} \cdot \frac{1}{1} = \frac{3a}{4b} \)
(common factor: 3a)

1 to 12: Reduce:
1. \( \frac{27}{36} = \frac{3}{4} \)
2. \( \frac{54}{68} = \frac{27}{34} \)
3. \( \frac{3}{5} \cdot \frac{6}{9} = \frac{2}{5} \)
4. \( \frac{6x}{19y} = \frac{3x}{9y} \)
5. \( 19a \cdot 2 \)
6. \( \frac{11x - 7y}{7} \)
7. \( \frac{5a + b}{4a + c} \)
8. \( \frac{x - 4}{x} \)
9. \( \frac{2(x + 4)(x - 5)}{(x - 5)(x - 4)} = \frac{2x + 2}{x + 1} = \frac{2x + 2}{x + 1} \)
10. \( \frac{x^2 - 9x}{x - 9} \)
11. \( \frac{8(x - 1)^2}{6(x^2 - 1)} \)
12. \( \frac{2x^2 - 2x - 1}{x^2 - 2x + 1} \)

example: \( \frac{3}{4} \) is equivalent to how many eights?
\( \frac{2}{4} = \frac{1}{2} \)
\( \frac{\frac{1}{2} \cdot \frac{3}{4}}{\frac{2}{3} \cdot \frac{3}{4}} = \frac{2}{3} \)
\( \frac{\frac{2}{3} \cdot \frac{3}{4}}{\frac{2}{3} \cdot \frac{3}{4}} = \frac{2}{3} \)

C. Equivalent fractions

example: \( \frac{3}{4} \) is equivalent to how many eights?
\( \frac{3}{4} = \frac{2}{4} = \frac{1}{2} \)
\( \frac{\frac{1}{2} \cdot \frac{3}{4}}{\frac{2}{3} \cdot \frac{3}{4}} = \frac{2}{3} \)
\( \frac{\frac{2}{3} \cdot \frac{3}{4}}{\frac{2}{3} \cdot \frac{3}{4}} = \frac{2}{3} \)

example: \( \frac{6}{5a} = \frac{5ab}{5a} \)
\( \frac{6}{5a} = \frac{b}{5a} = \frac{6}{5a} \)

example: \( \frac{2x + 2}{x + 1} = \frac{2x + 2}{x + 1} = \frac{2x + 2}{x + 1} \)

How to get the lowest common denominator (LCD) by finding the least common multiple (LCM) of all denominators:

1 to 14: Simplify:
1. \( \frac{14x}{6} \cdot \frac{2x}{7} = \frac{2x}{2} \)
2. \( \frac{x^2 - 3x}{x - 4} = \frac{2x - 6}{x - 4} \)

B. Evaluation of fractions

example: If \( a = -1 \) and \( b = 2 \), find the value of \( \frac{a + 3}{2b - 1} \)
Substitute: \( \frac{a + 3}{2b - 1} = \frac{-1 + 3}{2 \cdot 2 - 1} = \frac{2}{3} \)

15 to 22: Find the value, given \( a = -1 \), \( b = 2 \), \( c = 0 \), \( x = -3 \), \( y = 1 \), \( z = 2 \):
15. \( \frac{b}{a} = \frac{2}{-1} = -2 \)
19. \( \frac{1x - 5y}{1y - 2x} = \frac{1(-3) - 5(1)}{1(1) - 2(-3)} = \frac{-3 - 5}{1 + 6} = \frac{-8}{7} \)
20. \( \frac{2b}{a} = \frac{2(2)}{-1} = -4 \)
21. \( \frac{b - 1}{z} = \frac{2 - 1}{2} = \frac{1}{2} \)
22. \( \frac{a - x}{b} = \frac{-1 - (-3)}{2} = \frac{2}{2} = 1 \)

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One of a series of worksheets designed to provide remedial practice. Coordinated with topics on diagnostic tests supplied by the Mathematics Diagnostic Testing Program, College Entrance Exams Board, 1976.
D. Adding and subtracting fractions:
if denominators are the same, combine the numerators:

\[ \frac{3x}{y} - \frac{2x}{y} = \frac{3x - 2x}{y} = \frac{x}{y} \]

34. \( \frac{1}{y} + \frac{2}{y} = \)
35. \( \frac{x}{x - 3} - \frac{x}{x - 3} = \)
36. \( \frac{b}{a} - \frac{a}{b} = \frac{b - a}{a} = \frac{a - b}{b} = \)
37. \( \frac{x^2 - 2x}{x^2 + 2x} = \)
38. \( \frac{3a}{b} + \frac{2}{b} - \frac{a}{b} = \)

If denominators are different, find equivalent fractions with common denominators, then proceed as before (combine numerators):

\[ \frac{a}{2} - \frac{a}{4} = \frac{a(2) - a(1)}{4} = \frac{2a - a}{4} = \frac{a}{4} \]
\[ \frac{3}{x} + \frac{1}{x + 2} = \frac{3(x + 2) + 1(x - 1)(x + 2)}{x(x + 2)} = \frac{3x + 6 + x - 1}{x(x + 2)} = \frac{4x + 5}{x(x + 2)} \]

39. \( \frac{a}{b} - \frac{1}{2a} = \)
40. \( \frac{3}{a} - \frac{2}{a} = \)
41. \( \frac{b}{a} - \frac{2}{b} = \)
42. \( \frac{2}{a} + \frac{2}{a} = \)
43. \( \frac{x}{x - 1} + \frac{1}{1 - x} = \)
44. \( \frac{3x - 2}{x + 2} = \)
45. \( \frac{2x}{x - 1} - \frac{1}{x - 1} = \)
46. \( \frac{4x}{x - 2} - \frac{1}{x - 2} = \)
47. \( \frac{2x}{x} = \)
48. \( \frac{3x}{2} = \)
49. \( \frac{2x}{x} = \)
50. \( \frac{2x}{x} = \)
51. \( \frac{2x}{x} = \)

E. Multiplying fractions: multiply the tops, multiply the bottoms, reduce if possible:

\[ \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16} \]
\[ \frac{3(x + 1)}{x - 2} \cdot \frac{x^2 - 1}{x - 1} = \frac{3x^2 + 6}{x - 1} \]
52. \( \frac{2}{3} \cdot \frac{3}{2} = \)
53. \( \frac{5}{4} \cdot \frac{5}{4} = \)
54. \( \frac{2}{x} \cdot \frac{2}{x} = \)
55. \( \frac{a}{b} \cdot \frac{a}{b} = \)
57. \( \frac{a^2}{b^2} \cdot \frac{a^2}{b^2} = \)

F. Dividing fractions: a nice way to do this is to make a compound fraction and then multiply the top and bottom (of the big fraction) by the LCD of both:

\[ \frac{a + b}{c} \cdot \frac{d}{e} = \frac{a + b}{c} \cdot \frac{d}{e} = \frac{a + b}{c} \cdot \frac{d}{e} = \frac{a + b}{c} \cdot \frac{d}{e} = \]
\[ \frac{7}{3} - \frac{3}{2} = \frac{7(2) - 3(3)}{3(2)} = \frac{14 - 9}{6} = \frac{5}{6} \]
\[ \frac{5x}{2x} \cdot \frac{2x}{2x} = \frac{5x}{2x} \cdot \frac{2x}{2x} = \frac{5x}{2x} \cdot \frac{2x}{2x} = \]

60. \( \frac{1}{x} = \)
61. \( \frac{x + 7}{x} = \)
62. \( \frac{2}{x} = \)
63. \( \frac{a}{b} = \)
64. \( \frac{a + b}{2} = \)
65. \( \frac{a - b}{2} = \)
66. \( \frac{a - b}{2} = \)

EXAMPLES:

1. \( \frac{1}{3} \)
2. \( \frac{2}{5} \)
3. \( \frac{3}{4} \)
4. \( \frac{4}{5} \)
5. \( \frac{5}{6} \)
6. \( \frac{6}{7} \)
7. \( \frac{7}{8} \)
8. \( \frac{8}{9} \)
9. \( \frac{9}{10} \)
10. \( \frac{10}{11} \)
11. \( \frac{11}{12} \)
12. \( \frac{12}{13} \)
13. \( \frac{13}{14} \)
14. \( \frac{14}{15} \)
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30. \( \frac{30}{31} \)
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32. \( \frac{32}{33} \)
33. \( \frac{33}{34} \)
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35. \( \frac{35}{36} \)
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67. \( \frac{67}{68} \)
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70. \( \frac{70}{71} \)
**INTERMEDIATE ALGEBRA READINESS TEST**

**Topic 7: Exponents and square roots**

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Positive integer exponents

\( a^b \) means use \( a \) as a factor \( b \) times. (\( b \) is the exponent or power of \( a \)).

**Example:**

\[ 2^5 \text{ means } 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \text{, and has value } 32. \]

1 to 11: Find the value:

1. \( 2^3 = 8 \)
2. \( 2^2 = 4 \)
3. \( -4^2 = -16 \)
4. \( (-4)^2 = 16 \)
5. \( 0^4 = 0 \)
6. \( 1^4 = 1 \)
7. \( (\frac{2}{3})^4 = \frac{16}{81} \)
8. \( (2.3)^3 = 12.167 \)
9. \( (\frac{1}{2})^2 = \frac{1}{4} \)
10. \( 2^{10} = 1024 \)
11. \( (-2)^9 = -512 \)
12. \( (2.3)^2 = 5.29 \)
13. \( (-1.1)^3 = -1.611 \)
14. \( 3^2 \cdot 2^3 = 72 \)

To compute with numbers written in scientific form, separate the parts, compute, then recombine.

**Example:**

\[ (3.14 \times 10^5)(2) = 6.28 \times 10^5 \]

**Example:**

\[ 4.28 \times 10^6 \]

15 to 18: Simplify:

15. \( 3^2 \cdot x^4 = 9x^4 \)
16. \( 2^4 \cdot b \cdot b \cdot b \cdot b = 16b^4 \)
17. \( b^2(-x)(-x)(-x) = b^3 \)
18. \( (-y)^4 = y^4 \)

B. Integer exponents

1. \( a^b \cdot a^c = a^{b+c} \)
2. \( \frac{a^b}{a^c} = a^{b-c} \)
3. \( (a^b)^c = a^{bc} \)
4. \( (ab)^c = a^c \cdot b^c \)
5. \( (a^b)^c = a^{bc} \)
6. \( a^0 = 1 \) (if \( a \) ≠ 0)
7. \( a^{-b} = \frac{1}{a^b} \)

C. Scientific notation

**Example:**

\[ 32800 = 3.28 \times 10^4 \]

19 to 28: Find \( x \):

19. \( 3^2 \cdot 2^3 = 2^{x-2} \)
20. \( 2^3 = 2^x \)
21. \( 3^4 = \frac{27}{3^x} \)
22. \( \frac{5^4}{5^2} = 5^x \)
23. \( (2.3)^4 = 2^x \)
24. \( 8 = 2^x \)
25. \( a^2 \cdot a = a^{x+2} \)
26. \( \frac{b^{10}}{b^5} = b^x \)
27. \( \frac{1}{x^4} = x^x \)
28. \( \frac{3y}{x^2} = 2 \)

29 to 33: Find the value:

29. \( 7^3 = 343 \)
30. \( 3^4 = 81 \)
31. \( 2^3 = 8 \)
32. \( 2^2 = 4 \)
33. \( 5^0 = 1 \)
34. \( (-3)^3 = -27 \)
35. \( x^2 \cdot x^{-3} = \frac{x}{x^3} \)
36. \( x^2 \cdot x^{-3} = \frac{x}{x^3} \)
37. \( x^2 \cdot x^{-3} = \frac{x}{x^3} \)
38. \( (a^2)^3 \cdot x^{-3} = \frac{x}{x^3} \)
39. \( (2^3)^2 = \frac{x}{x^3} \)
40. \( (3x)^2 = \frac{x}{x^3} \)
41. \( (-2x^2)^3 = \frac{x}{x^3} \)

42. **Note that scientific form always looks like \( a \times 10^n \) where \( 1 \leq a < 10 \) and \( n \) is an integer power of 10.**

43. \( 32 \times 10^3 = 3.2 \times 10^4 \)
44. \( 5.00 \times 10^{-2} = 0.05 \)
45. \( -1.2 \times 10^{-3} = -0.0012 \)

46 to 48: Write in scientific notation:

46. \( 1.403 \times 10^3 = 1.403 \times 10^3 \)
47. \( -9.11 \times 10^{-2} = -0.0911 \)
48. \( 4.10 \times 10^{-6} = 4.10 \times 10^{-6} \)

49 to 56: Write answer in scientific notation:

49. \( 10^{0.0} = 1 \)
50. \( 10^{-10} = 1 \)
51. \( 1.86 \times 10^4 = 1.86 \times 10^4 \)
52. \( 3.6 \times 10^3 = 3.6 \times 10^3 \)
53. \( 2.6 \times 10^6 = 2.6 \times 10^6 \)
54. \( (4 \times 10^3)^2 = 16 \times 10^6 \)
55. \( (2.5 \times 10^2)^{-1} = 4 \times 10^{-4} \)
56. \( (-2.22 \times 10^3)(4.1 \times 10^7) = -8.2 \times 10^{10} \)

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### D. Simplification of Square Roots

\[ \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \text{ if } a \text{ and } b \text{ are both non-negative (} a \geq 0 \text{ and } b \geq 0). \]

**Example:**
- \( \sqrt{32} = \sqrt{16 \cdot 2} = 4 \sqrt{2} \)
- \( \sqrt{75} = \sqrt{3 \cdot 25} = 5 \sqrt{3} \)
- If \( x \geq 0 \), \( \sqrt{x^2} = x \)
- If \( x < 0 \), \( \sqrt{x^2} = |x| \)

Note: \( \sqrt{a} = b \) means (by definition) that
1) \( b^2 = a \), and
2) \( b \geq 0 \)

### E. Multiplying Square Roots

**Example:**
- \( \sqrt{5} \cdot \sqrt{2} = \sqrt{10} \)
- \( \sqrt{12} \cdot \sqrt{8} = \sqrt{96} \)
- \( (\sqrt{2})(\sqrt{3}) = \sqrt{6} \)

74 to 79: Simplify:
- \( \sqrt{3} \cdot \sqrt{3} = \sqrt{9} = 3 \)
- \( \sqrt{12} \cdot \sqrt{2} = \sqrt{24} \)
- \( \sqrt{2} \cdot \sqrt{3} = \sqrt{6} \)

80 to 81: Find the value of \( x \):
- \( \sqrt{4} \cdot \sqrt{9} = \sqrt{x} \)
- \( 3 \sqrt{2} \cdot \sqrt{5} = 3 \sqrt{10} \)

### F. Dividing Square Roots

**Example:**
- \( \frac{\sqrt{5}}{\sqrt{2}} = \sqrt{\frac{5}{2}} = \frac{\sqrt{10}}{4} \)

82 to 86: Simplify:
- \( \sqrt{3} \div \sqrt{4} = \frac{\sqrt{3}}{2} \)
- \( \sqrt{\frac{8}{2}} = \sqrt{4} \)
- \( \sqrt{6} \div \sqrt{4} = \frac{\sqrt{3}}{2} \)
- \( \sqrt{16} = 4 \)

If a fraction has a square root on the bottom, it is sometimes desirable to find an equivalent fraction with no root on the bottom. This is called rationalizing the denominator.

### G. Dividing Square Roots

**Example:**
- \( \frac{\sqrt{3} \cdot \sqrt{2}}{\sqrt{3} \div \sqrt{2}} = \frac{\sqrt{6}}{2} \)

87 to 94: Simplify:
- \( \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2} \)
- \( \sqrt{\frac{16}{9}} = \frac{4}{3} \)
- \( \sqrt{\frac{25}{16}} = \frac{5}{4} \)

Answers:
- 1. 8
- 2. 9
- 3. -8
- 4. 16
- 5. 0
- 6. 1
- 7. 16/61
- 8. \( \frac{2}{5} \)
- 9. \( \frac{5}{12} \)
- 10. \( \frac{1}{12} \)
- 11. \( \frac{2}{11} \)
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INTERMEDIATE ALGEBRA READINESS TEST

Topic 3: Geometric measurement

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Intersecting lines and parallel lines: If two lines intersect as shown, adjacent angles add to 180°. For example, a + d = 180°. Non-adjacent angles are equal: for example, a = c.

If two lines, a and b, are parallel and cut by a third line c, forming angles w, x, y, z as shown, then x = z, w = y, w + y = 180°, so x + y = 180°.

Example: If a = 5x and b = x, find the value of c.

b = a, so b = 5x.
a + b = 180°, so
5x + x = 180°, giving x = 30°, or x = 35°.

Thus a = 150°.

B. Formulas for perimeter P and area A of triangles, squares, rectangles, and parallelograms

Rectangle: base b, altitude a

P = 2b + 2h
A = bh

A square is a rectangle with all sides equal, so the formulas are the same (and simpler if the side length is s):

P = 4s
A = s²

Example: Square with side 11 cm has P = 46 + 11 = 46 + 36 cm
A = s² = 11² = 121 cm² (sq. cm)

A parallelogram has base b and height h = A = bh

If the other side length is a, then P = 2a + 2b

Example: Parallelogram has sides 4 and b, and 5 is the length of the altitude perpendicular to the side 4.

P = 2a + 2b = 6 + 8 = 12 + 8 = 20 units
A = bh = 4 x 5 = 20 sq. units

In a triangle with side lengths a, b, c and h is the altitude to side b:

P = a + b + c
A = bh/2

Example: P = 2 + 3 + 4 = 9 units
A = 3 x h/2

6 to 13: Find P and A for each of the following figures:

6. Rectangle with sides 5 and 10.


8. Square with side 3 m.


11. Parallelogram, all sides 12, altitude 5.

12. Triangle with sides 5, 12, 13, and 5 is the height on side 12.

13. The triangle shown:

Example: P = 10 + 10 + 12 = 32 units
A = 12 x 5/2 = 30 sq. units

C. Formulas for circle area A and circumference C

A circle with radius r (and diameter d = 2r) has area (circumference) C = πr² (C = 2πr)

If a piece of wire is bent into a circular shape, the circumference is the length of wire.

Example: A circle with radius 7 and exact circumference C = 2πr = 70 units.
If r is approximated by 5, C = 2(5) = 10π units.

Example: P = 60π units.

Example: P = 314 units, approximate C = 126(314) = 39364 units.

The area of a circle A = πr²

Example: If r = 5,
A = 314 = 25π² = 625 sq. units

14 to 16: Find C and A for each triangle:

14. a = 14 units
15. r = 5 units
16. d = 5 km

Example: The measures of angles C and A:

Example: Find measures of angles C and A:

Example: The measure of an angle C is marked to show its measure is 90°.

Example: C + 2C = 26 + 90 = 116, so

A = 180 - 126 = 54°
25 to 29: Given two angles of a triangle, find the measure of the third angle:

25. $30^\circ$, $60^\circ$

26. $15^\circ$, $36^\circ$

27. $90^\circ$, $17^\circ$

28. $82^\circ$, $82^\circ$

29. $68^\circ$, $44^\circ$

P. Isosceles triangles

An isosceles triangle is defined to have at least two sides with equal measure. The equal sides may be marked:

or the measures may be given:

30 to 35: Is the triangle isosceles?

30. Sides 3, 4, 5

31. Sides 7, 4, 7

32. Sides 6, 8, 8

33. The angles which are opposite the equal sides also have equal measures (and all three angles add up to $180^\circ$).

example: Find the measures of $\angle A$ and $\angle C$, given $\angle B = 65^\circ$.

$\angle A + \angle B + \angle C = 180^\circ$

$\angle A = \angle B = 65^\circ$, so $\angle C = 50^\circ$

36. Find measures of $\angle A$ and $\angle C$, if $\angle B$ is $30^\circ$.

37. Find measures of $\angle B$ and $\angle C$, if $\angle A$ is $30^\circ$.

38. Find measure of $\angle A$.

39. If the angles of a triangle are $30^\circ$, $60^\circ$, and $90^\circ$, can it be isosceles?

40. If two angles of a triangle are $45^\circ$ and $60^\circ$, can it be isosceles?

If a triangle has equal angles, the sides opposite these angles also have equal measures.

example: Find the measures of $\angle B$, $\angle D$, and $\angle A$, given this figure, and $\angle C = 40^\circ$.

$\angle B = 70^\circ$ (because all angles add up to $180^\circ$)

Since $\angle A = \angle B$, $\angle A = 70^\circ$.

$\angle D$ can be found with trigonometry.

41. Can a triangle be isosceles and have a $90^\circ$ angle?

42. Given $\angle D = \angle B = 40^\circ$ and $DF = 6$. Find the measure of $\angle D$ and length of $FB$.

O. Similar triangles: If two angles of one triangle are equal to two angles of another triangle, then the triangles are similar.

example: $\triangle ABC$ and $\triangle DEF$ are similar.

The pairs of corresponding sides are $AB$ and $DE$, $BC$ and $EF$, and $AC$ and $FD$.

43. Name two similar triangles and list the pairs of corresponding sides.

If two triangles are similar, any two corresponding sides have the same ratio (fraction value):

example: the ratio $a$ to $x$, or $\frac{a}{x}$, is the same as $\frac{b}{y}$.

Thus, $\frac{a}{b} = \frac{x}{y}$.

Each of these equations is called a proportion.

44. to 45: Write proportions for the two similar triangles.

46. example: Find $x$:

Write and solve a proportion:

$x = \frac{2}{3}$, so $2x = 15$, $x = \frac{15}{2}$

47. to 48: Find $x$:

49. Find $x$ and $y$.

50. Find $x$ and $y$.

E. Pythagorean theorem

In any triangle with a $90^\circ$ (right) angle, the sum of the squares of the legs equals the square of the hypotenuse. (The legs are the two shorter sides; the hypotenuse is the longest side.) If the legs have length $a$ and $b$, and the hypotenuse length is $c$, then

$a^2 + b^2 = c^2$ (In words, "In a right triangle, leg squared plus leg squared equals hypotenuse squared.")

example: A right triangle has hypotenuse 5 and one leg 3. Find the other leg.

Since $a^2 + b^2 = c^2$, $3^2 + x^2 = 5^2$.

$x^2 = 25 - 9 = 16$

$x = \sqrt{16} = 4$

51 to 54: Each line of the chart lists two sides of a right triangle. Find the length of the third side:

<table>
<thead>
<tr>
<th>leg</th>
<th>leg</th>
<th>hypotenuse</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>$\sqrt{2}$</td>
<td>$\sqrt{3}$</td>
<td></td>
</tr>
</tbody>
</table>

55 to 56: Find $x$:

57.
A. Arithmetic, percent, and average

1. What is the number, which when multiplied by 35, gives 35256?
2. If you square a certain number, you get 99. What is the number?
3. What is the power of 36 that gives 362?
4. Find the square root of 36.
5. 55 is what percent of 68?
6. What percent of 55 is 88?
7. 45 is 80% of what number?
8. What is 5% of $7000?

9. If you get 36 on a 100-question test, what percent is this?
10. The 3200 people who vote in an election are 40% of the people registered to vote. How many are registered?

11 to 13: Your wage is increased by 25%, then the new amount is cut by 20% (of the new amount).
11. Will this result in a wage which is higher than, lower than, or the same as the original wage?
12. What percent of the original wage is this final wage?
13. If the above steps were reversed (20% cut followed by 25% increase), the final wage would be what percent of the original wage?

14 to 16: If $A$ is increased by 25%, it equals $B$.
14. Which is larger, $B$ or the original $A$?
15. $B$ is what percent of $A$?
16. $A$ is what percent of $B$?
17. What is the average of 87, 36, 48, 59, and 95?
18. If two test scores are 65 and 60, what minimum score on the next test would be needed for an overall average of 80?
19. The average height of 69 people is 68 inches. What is the new average height if a 78-inch person joins the group?

B. Algebraic substitution and evaluation

20 to 22: A certain TV uses 75 watts of power and operates on 120 volts.
20. Find how many amps of current it uses, from the relationship: volts times amps equals watts.
21. 1000 watts = 1 kilowatt (kW). How many kilowatts does the TV use?
22. kW times hours = kilowatt-hours (kWh). If the TV is on for six hours a day, how many kWh of electricity are used?

23. If the set is on for six hours every day of a 30-day month, how many kWh are used for the month?
24. If the electric company charges $4 per kWh, what amount of the month's bill is for TV power?
25 to 28: A plane has a certain speed in still air, where it goes 1350 miles in three hours.
25. What is its (still air) speed?
26. How far does the plane go in 5 hours?
27. How far does it go in $x$ hours?
28. How long does it take to fly 2000 mi.?
29. How long does it take to fly $y$ mi.?
30. If the plane flies against a 50 mph headwind, what is its ground speed?
31. If the plane flies against a headwind of $x$ mph, what is its ground speed?
32. If it has fuel for 7.5 hours of flying time, how far can it go against the headwind of 50 mph?
33. If the plane has fuel for $t$ hours of flying time, how far can it go against the headwind of $x$ mph?

C. Ratio and proportion

34 to 35: $x$ is to $y$ as 3 is to 5.
34. Find $y$ when $x$ is 7.
35. Find $x$ when $y$ is 7.
35 to 37: $x$ is proportional to $y$, and $F = 56$ when $x = 14$.
36. Find $x$ when $F = 112$.
37. Find $F$ when $x = 112$.
38 to 39: Given $3x = 4y$.
38. Write the ratio $x : y$ as the ratio of two integers.
39. If $x = 3$, find $y$.

40 to 41: $x$ and $y$ are numbers, and two $x$'s equal three $y$'s.
40. Which of $x$ or $y$ must be larger?
41. What is the ratio of $x$ to $y$?
42 to 44: Half of $x$ is the same as one-third of $y$.
42. Which of $x$ and $y$ is the larger?
43. Write the ratio $x : y$ as the ratio of two integers.
44. How many $x$'s equal 30 $y$'s?
48. 2/3 of 1/6 of 3/4 of a number is 12. What is the number?
49. Half the square of a number is 10. What is the number?
50. 61 is the square of twice what number?
51. Given a positive number \( x \). Two times a positive number \( y \) is at least four times \( x \). How small can \( y \) be?
52. Twice the square root of half of a number is 2x. What is the number?
53 to 55: A gathering has twice as many women as men. \( W \) is the number of women and \( M \) is the number of men.
53. Which is correct: \( 2M = W \) or \( \frac{M}{W} = \frac{1}{2} \) ?
54. If there are 12 women, how many men are there?
55. If the total number of men and women present is 54, how many of each are there?
56. $12,000 is divided into equal shares. Bob gets four shares, Bill gets three shares, and Ben gets the one remaining share. What is the value of one share?

E. Problems leading to two linear equations
57. Two science fiction coins have values \( x \) and \( y \). Three \( x \)'s and five \( y \)'s have a value of $756, and one \( x \) and two \( y \)'s have a value of $276. What is the value of each?
58. In mixing \( x \) gm of 3% and \( y \) gm of 8% solutions to get 10 gm of 5% solution, these equations are used: 
\[
0.03x + 0.08y = 0.05(10), \quad \text{and} \quad x + y = 10
\]
How many gm of 3% solution are needed?

F. Geometry
59. Point \( X \) is on each of two given intersecting lines. How many such points \( X \) are there?
60. On the number line, points \( P \) and \( Q \) are two units apart. \( Q \) has coordinate \( x \). What are the possible coordinates of \( P \)?

61 to 62:
61. If the length of chord \( AB \) is \( x \) and the length of chord \( GB \) is 16, what is \( AC \)?
62. If \( AC = x \) and \( GB = x \), how long is \( AB \) (in terms of \( x \) and \( z \)?)

63 to 64: The base of a rectangle is three times the height.
63. Find the height if the base is 20.
64. Find the perimeter and area.