5.7 Introduction to Square Roots

The Square of a Number

The number $x^2$ is called the square of the number $x$.

EX) $9^2 = 9 \cdot 9 = 81$, the number 81 is the square of the number 9.

$$(-4)^2 = (-4)(-4) = 16,$$ the number 16 is the square of the number $-4$.

The Square ROOT of a Number

If $b = a^2$, then $a$ is called the square root of the number $b$.

So, if $b = a^2$, then $\sqrt{b} = a$.

EX) Since $81 = 9^2$, then $\sqrt{81} = 9$. 9 is the square root of 81.

Since $25 = 5^2$, then $\sqrt{25} = 5$. 5 is the square root of 25.

Example 1: Find the square roots of 49.

Solution: There are TWO square roots of 49, a positive number and a negative number. To find our solutions, we need to think about what number(s) squared equals 49, $(?)^2 = 49$. Since, $7^2 = 49$, we know one of our solutions is 7. But remember we should have a positive number and a negative number for our solutions. It is also true that $(-7)^2 = 49$. Therefore, $-7$ is our other solution.

So, again, since $7^2 = 49$ and $(-7)^2 = 49$, we have that the square roots of 49 are $7$ and $-7$.

You Try It 1: Find the square roots of 256.

Example 2: Find the square roots of 196.

Solution: There are TWO square roots of 196, a positive number and a negative number.

So, since $14^2 = 196$ and $(-14)^2 = 196$, we have that the square roots of 196 are $14$ and $-14$.

You Try It 2: Find the square roots of 625.
Example 3: Find the square roots of 0.

Solution: There only ONE square root of 0.
Since $0^2 = 0$, we have that the square root of 0 is $0$. There is no negative 0, which is why we only have one solution.

You Try It 3: Find the square roots of 9.

Example 4: Find the square roots of $-25$.

Solution: If we try to think of what number squared equals $-25$, we will notice that no number squared will equal a negative number, $a^2 \neq -b$. Therefore $-25$, has no square roots.

You Try It 4: Find the square roots of $-81$.

Radical Notation

In the expression, $\sqrt{9}$, the symbol $\sqrt{}$ is called a radical and the number within the radical, in this case the 9, is called the radicand.

Note: When using radical notation, you will only have one solution for each radical.
For example: $\sqrt{9} = 3$. So when it says, simplify $\sqrt{9}$, you will only write 3 as your answer.
Example 5: Simplify. 

a) \( \sqrt{121} \)  

b) \(-\sqrt{625}\)  

c) \(\sqrt{0}\)

Solution: 

a) We know \(11^2 = 121\), so \(\sqrt{121} = 11\).

b) \(-\sqrt{625}\). The negative in front of the radical has nothing to do with the radical, it will simply carry over into your answer. We know \(25^2 = 625\), so \(-\sqrt{625} = -25\). Hence, \(-\sqrt{625} = -25\).

c) We know \(0^2 = 0\), so \(\sqrt{0} = 0\).

You Try It 5: Simplify. 

a) \(\sqrt{144}\)  

b) \(-\sqrt{324}\)
Example 6: Simplify.  a) \(-\sqrt{25}\)  

Solution: \(-\sqrt{25}\), We know \(5^2 = 25\), so \(-\sqrt{25} = -5\). Hence, \(-\sqrt{25} = -5\).

b) \(\sqrt{-25}\), There is no real number that you can square that will give you \(-25\). Hence, in this course, any time you see a negative INSIDE the radical, your answer will be “no real solution”.

You Try It 6: Simplify.  a) \(-\sqrt{36}\)  

b) \(\sqrt{-36}\)

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Order of Operations

1) Perform all operations inside the grouping symbols. Start with the innermost grouping and perform all operations inside until you are left with a single number.

2) *Evaluate any exponents and radicals in the expression.

3) Moving from left to right, perform any multiplications or divisions in the order in which they appear.

4) Moving from left to right, perform any additions or subtractions in the order in which they appear.

*Note: Now that we have learned about radicals, radicals have been added to the 2nd step in the Order of Operations.

Example 7: Simplify. \(-3\sqrt{9} + 12\sqrt{4}\)

Solution: \(-3\sqrt{9} + 12\sqrt{4}\)

1) There are no operations to simplify inside grouping symbols.

2) There are radicals so we will simplify them in this step.

\[-3\sqrt{9} + 12\sqrt{4} = -3(3) + 12(2)\]

3) Next we will do the multiplications.

\[-3(3) + 12(2) = -9 + 24\]

4) Last we do the addition.

\[-9 + 24 = 15\]
You Try It 7: Simplify. $2\sqrt{4} - 3\sqrt{9}$

Example 8: Simplify. $-2 - 3\sqrt{36}$

Solution: $-2 - 3\sqrt{36}$

1) There are no operations to simplify inside grouping symbols.
2) There is a radical so we will simplify it in this step.
   
   $-2 - 3\sqrt{36} = -2 - 3(6)$
   
   $6$
3) Next we will do the multiplication.
   
   $-2 - 3(6) = -2 - 18$
   
   $-18$
4) Last we do the addition.
   
   $-2 - 18 = -20$

You Try It 8: Simplify. $5 - 8\sqrt{169}$

Example 9: Simplify. a) $\sqrt{9} + 16$   
   b) $\sqrt{9} + \sqrt{16}$

Solution: a) $\sqrt{9} + 16$

1) The radical is a grouping symbol. On this step we will simplify inside the grouping symbol.
   
   $\sqrt{9} + 16 = \sqrt{25}$
   
   $25$
2) There is a radical so we will simplify it in this step.
   
   $\sqrt{25} = 5$

And we’re down to one number so we are done. $\sqrt{9} + 16 = 5$

b) $\sqrt{9} + \sqrt{16}$

1) There are no operations to simplify inside grouping symbols.
2) There are radicals so we will simplify them in this step.
   
   $\sqrt{9} + \sqrt{16} = 3 + 4$
   
   $3$

3) There are no multiplications or divisions so we skip this step.
4) Last we do the addition.
   
   $3 + 4 = 7$

You Try It 9: Simplify. a) $\sqrt{25} + \sqrt{144}$   
   b) $\sqrt{25} + \sqrt{144}$

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Example 10: Simplify. a) \(\sqrt{\frac{4}{9}}\)  b) \(-\sqrt{0.49}\)

Solution: a) \(\sqrt{\frac{4}{9}}\), So we are looking for what number squared is \(\frac{4}{9}\). It might be easier to think of the numerator and denominator separately. In other words, what number squared becomes 4 and what number squared becomes 9. It makes it easier to see that our answer is \(\sqrt{\frac{4}{9}} = \frac{2}{3}\).

b) \(-\sqrt{0.49}\), If we ignore the decimal for a bit, we can ask ourselves what number squared is 49. We then know we should see a 7 in our answer. Now looking at the decimal, we see that there are two digits behind the decimal point. Remember that when you square a number you are multiplying it by itself. So if we are finding the square root (which is the opposite of squaring a number) we want to divide the number of decimal places by half. Therefore, if there are two decimal places our answer for the square root will only have one decimal place.

Hence, \(-\sqrt{0.49} = -0.7\), you can see that this is true either by plugging \(-\sqrt{0.49}\) into your calculator or by seeing that \((0.7)^2 = 0.49\).

So, \(-\sqrt{0.49} = -0.7\)

You Try It 10: Simplify. a) \(\sqrt{\frac{25}{49}}\)  b) \(\sqrt{0.36}\)

Estimating Square Roots

Example 11: Estimate the given square root. \(\sqrt{24}\)

a) Determine the two integers that the square root lies between.

b) Use your calculator to find the approximation of the square root to the nearest tenth.

Solution: a) If you look at all the perfect square numbers (1, 4, 9, 16, 25, 36, 49, 64, 81, 100, …) you’ll see that 24 lies in between the perfect square numbers 16 and 25. Since, \(\sqrt{16} = 4\) and \(\sqrt{25} = 5\), the \(\sqrt{24}\) must lie between integers 4 and 5. It is probably closer to the integer 5 since 24 is closer to 25. Hence, the \(\sqrt{24}\) is in between integers \(4\) and \(5\).  

b) If you use your calculator, you will get \(\sqrt{24} = 4.898979486\). Rounding we have \(\sqrt{24} \approx 4.9\).

You Try It 11: Estimate the given square root. \(\sqrt{10}\)

a) Determine the two integers that the square root lies between.

b) Use your calculator to find the approximation of the square root to the nearest tenth.