When factoring, root extraction cannot be used to solve a quadratic equation; another method can be employed called *completing the square*.

**Example 1** Consider the equation \( x^2 + 7x - 9 = 0 \).

1. Isolate the \( x \)-terms by adding 9 to both sides of the equation.
   \[
   x^2 + 7x = 9
   \]
2. Take \( \frac{1}{2} \) of the coefficient in front of the \( x \)-terms and square it.
   \[
   \left( \frac{1}{2} \cdot 7 \right)^2 = \left( \frac{7}{2} \right)^2 = \frac{49}{4}
   \]
3. Add this constant to both sides of the equation.
   \[
   x^2 + 7x + \frac{49}{4} = 9 + \frac{49}{4}
   \]
4. Left-hand side should be the perfect square of a binomial.
   \[
   \left( x + \frac{7}{2} \right)^2 = 9 + \frac{49}{4}
   \]

**REMARK:** The constant in the binomial will be half the coefficient of the \( x \)-term.

5. Combine the terms on the right side of the equation.
   \[
   \left( x + \frac{7}{2} \right)^2 = \frac{36}{4} + \frac{49}{4}
   \]
   \[
   \left( x + \frac{7}{2} \right)^2 = \frac{85}{4}
   \]
6. Take the square root from both sides.
   \[
   x + \frac{7}{2} = \pm \sqrt{\frac{85}{4}} = \pm \frac{\sqrt{85}}{2}
   \]
7. Solve for \( x \) by subtracting \( \frac{7}{2} \) from both sides.
   \[
   x = \frac{-7}{2} \pm \frac{\sqrt{85}}{2} \text{ or } x = \frac{-7 \pm \sqrt{85}}{2}
   \]

Therefore, the solution set is: \( \left\{ \frac{-7 + \sqrt{85}}{2}, \frac{-7 - \sqrt{85}}{2} \right\} \text{ or } \left\{ \frac{-7 \pm \sqrt{85}}{2} \right\} \)
Example 2  Consider the equation $-4x + 3x^2 = 11$.

1. Rearrange the $x$-terms in descending order. The constant is already on the other side.
   \[ 3x^2 - 4x = 11 \]

2. Divide both sides by 3. It is necessary that the coefficient in front of the $x$-term be a positive 1.
   \[ x^2 - \frac{4}{3}x = \frac{11}{3} \]

3. Take $\frac{1}{2}$ of the middle term and square it.
   \[ \left( \frac{1}{2} \cdot \left( -\frac{4}{3} \right) \right)^2 = \left( -\frac{2}{3} \right)^2 \]
   \[ = \frac{4}{9} \]

4. Add this constant to both sides of the equation and simplify.
   \[ x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{11}{3} + \frac{4}{9} \]
   \[ x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{33}{9} + \frac{4}{9} \]
   \[ x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{37}{9} \]

5. Factor the left-hand side.
   \[ (x - \frac{2}{3})^2 = \frac{37}{9} \]

6. Take the square root of both sides.
   \[ x - \frac{2}{3} = \pm \sqrt{\frac{37}{9}} \]
   \[ = \pm \frac{\sqrt{37}}{3} \]

7. Solve for $x$ by adding $\frac{2}{3}$ to both sides.
   \[ x = \frac{-7}{2} \pm \frac{\sqrt{37}}{2} \]
   \[ = \frac{-7 \pm \sqrt{37}}{2} \]

Therefore, the solution set is: \( \left\{ \frac{2 \pm \sqrt{37}}{3} \right\} \)
Example 3  Solve \(-4x^2 + 5x + 7 = 0\).

\[-4x^2 + 5x = -7\]

\[x^2 - \frac{5}{4}x = \frac{7}{4}\]

\[x^2 - \frac{5}{4}x + \frac{25}{64} = \frac{7}{4} + \frac{25}{64}\]

\[\left(x - \frac{5}{8}\right)^2 = \frac{112}{64} + \frac{25}{64}\]

\[\left(x - \frac{5}{8}\right)^2 = \frac{137}{64}\]

\[x - \frac{5}{8} = \pm \frac{\sqrt{137}}{8}\]

\[x = \frac{5}{8} \pm \frac{\sqrt{137}}{8}\]

The solutions are: \[\left\{\frac{5 \pm \sqrt{137}}{8}\right\}\]

Problem Set A  Supply the missing term needed to complete the square.

1. \(x^2 + 5x + \underline{\quad}\)  
2. \(y^2 - 6y + \underline{\quad}\)  
3. \(2x^2 - 18x + \underline{\quad}\)

4. \(-d^2 + 2d + \underline{\quad}\)  
5. \(-4a^2 - a + \underline{\quad}\)  
6. \(10x^2 - 11x + \underline{\quad}\)

7. \(z^2 - 3z + \underline{\quad}\)  
8. \(10b - 6b^2 + \underline{\quad}\)  
9. \(-x^2 + 12x + \underline{\quad}\)

10. \(5u + 3u^2 + \underline{\quad}\)

Answers A

1. \(\frac{25}{4}\)  
2. 9  
3. \(\frac{81}{4}\)  
4. 1  
5. \(\frac{1}{64}\)  
6. \(\frac{121}{400}\)  
7. \(\frac{9}{4}\)  
8. \(\frac{25}{9}\)  
9. 36  
10. \(\frac{25}{36}\)

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**Problem Set B**  Solve by completing the square.

1. \(x^2 + 6x + 8 = 0\)  
2. \(x^2 + 7x = 8\)  
3. \(2x^2 - 6x - 5 = 0\)  
4. \(x^2 - 5x = -3\)  
5. \(3x^2 - 5x + 1 = 0\)  
6. \(6x^2 - 1 = -x\)  
7. \(7x + 4 = -2x^2\)  
8. \(x^2 + 3x - 10 = 0\)  
9. \(4x^2 + 9x = 0\)  
10. \(x^2 - 11x + 30 = 0\)  
11. \(2x^2 - 7x - 15 = 0\)  
12. \(6x^2 + x - 5 = 0\)  
13. \(6x^2 + 5x - 6 = 0\)  
14. \(12x^2 - 4x - 5 = 0\)  
15. \(10x^2 - 4x - 1 = 0\)  
16. \(4x^2 + 9x - 9 = 0\)  
17. \(4x^2 + 5x - 2 = 0\)  
18. \(2x^2 + 7x = 3\)  
19. \(3x^2 = 6x + 2\)  
20. \(3x^2 = 2x + 3\)

**Answers B**

1. \(\{-4, -2\}\)  
2. \(\{-8, 1\}\)  
3. \(\{\frac{6 \pm \sqrt{76}}{4}\}\)  
4. \(\left\{\frac{5 \pm \sqrt{13}}{2}\right\}\)  
5. \(\left\{\frac{5 \pm \sqrt{13}}{6}\right\}\)  
6. \(\left\{-\frac{1}{2}, \frac{1}{3}\right\}\)  
7. \(\left\{-\frac{7 \pm \sqrt{17}}{4}\right\}\)  
8. \(\{-5, 2\}\)  
9. \(\{0, -\frac{9}{4}\}\)  
10. \(\{5, 6\}\)  
11. \(\left\{-\frac{3}{2}, 5\right\}\)  
12. \(\left\{\frac{5}{6}, -1\right\}\)  
13. \(\left\{-\frac{3}{2}, \frac{2}{3}\right\}\)  
14. \(\left\{\frac{5}{6}, \frac{1}{2}\right\}\)  
15. \(\left\{\frac{2 \pm \sqrt{14}}{10}\right\}\)  
16. \(\left\{-3, \frac{3}{4}\right\}\)  
17. \(\left\{-\frac{5 \pm \sqrt{57}}{8}\right\}\)  
18. \(\left\{-\frac{7 \pm \sqrt{73}}{4}\right\}\)  
19. \(\left\{\frac{3 \pm \sqrt{15}}{3}\right\}\)  
20. \(\left\{\frac{1 \pm \sqrt{10}}{3}\right\}\)