Welcome to Calculus

Section 2.6 Related Rates
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- Warm up
  1. Find $dy/dt$ (implicitly) if $y^2 - 4y - t = -4$
  2. Find $dy/dt$ from #1, if $y = 4$
  3. Find $dy/dt$ (implicitly) if $y^2 - 4y - t^2 = -4$
  4. Suppose $x$ and $y$ are both differentiable functions of $t$ and if $y = x^2 + 3$. Solve $dy/dt$ (in terms of variable $x$ and $dx/dt$.)
  5. From #4 Find $dy/dt$ given $x = 1$ and $dx/dt = 2$
  6. If $V = \pi r^2 h$, where $r$ and $h$ both are differentiable function of $t$. Find $dV/dt$ implicitly.
Section 2.6 Overview

- **Objective:** After this lesson, you will be able to:
  - Find a related rate by applying implicit derivative and chain rule.
  - Use related rates to solve real-life problems

- Examples of application:
  - When the water is drained out of a conical tank
  - When the oil spills, it creates a circular pool
  - When you drop a pebble into a calm pond, the water ripples in the form of concentric circles
  - When you watching a train passing by, your eye moves with the train
  - When you pump the air into a spherical balloon with certain rate cubic feet per minute
  - When the ground radar tracks the coming airplane
  - When a television camera taking picture of the shuttle launches.

- The rate change is like we can freeze the time at that moment and finding the rate at that moment.
2.6a Find rate change

- Example 1: If $y = x^2 - 3x + 2$,
  
  a. find $dy/dt$ when $x = 4$ and $dx/dt = 3$
  b. find $dx/dt$ when $x = 3$ and $dy/dt = 2$

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- Example 2: if $x^2 + y^2 = 25$
  
  a. Find $dy/dt$ when $x = 3$, $y = 4$, and $dx/dt = 8$
  b. Find $dx/dt$ when $x = 4$, $y = 3$, and $dy/dt = -2$
Mathematical model of rate change

Suppose you drive a car from point A to B, the distance your traveled is $x$, and with the speed of 50 miles per hour, how would you write in mathematical model for the rate change of distance?

Suppose the water is being pumped into a swimming pool at a rate of 10 cubic meters per hour, what is the mathematical model for the change of volume of the water in swimming pool?

Suppose a gear is revolving at a rate of 25 revolutions per minute, what is the mathematical model for the rate change of the angle of the gear? What is one revolution in radians? A gear is revolving with an angle $\theta$
Rate change of Distance

- Example 3: a point is moving along the graph of the given function such that $dx/dt = 2\text{cm per second}$. Find $dy/dt$ for the given values of $x$
  a. $y = x^2 - 1$, when $x = -2$
  b. $y = \frac{1}{1+x^2}$ when $x = -2$
  c. $y = \sin x$ when $x = \pi/6$, or $x = \pi/4$, or $x = 0$.
  d. Find the rate change of the distance between the origin and a moving point on the graph $y = \sin x$, if $dx/dt = 2$ centimeter per second.

- If you have a linear function $y = ax + b$. If $x$ changes at a constant rate with respect to time, does $y$ change at a constant rate with respect to time? If so, does $y$ change at the same rate as $x$?
Solving problems with areas

Example 1: A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. If the radius of outside ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area $A$ of the disturbed water changing?

Notations:

- Area is unit square
- Rate of area (increasing or decreasing) with respect to time is denoted by $\frac{dA}{dt} = \text{unit/time (per second, per minute or per hour)}$

Solution:

- This is a circle area, thus $A$ (area) = $\pi r^2$, i.e. $A$ depends on $r$. The value of $r$ depends on time. Thus both $A$ and $r$ are function of $r$ implicitly. Since derivative is rate change, and we are interested the change with respect to time, thus we need to take derivative implicitly.
Example 2: Oil Spill Problem

- An oil spill created a circular pool whose area increased at a rate of $30\pi m^2$/minutes. How fast was the radius of the pool increasing when the radius was 5 meters?

- Solution:
Homework Assignment for 2.6a

- Homework for 2.6a, #1-7 odd, 11-17 odd.