Let $D =$ the number of dates in the pile

If $2000 < D \leq 3000$ then the camel must make 3 trips to move the pile a distance, $k_1$ miles, 2 round trips and 1 one-way trip. Thus after moving the pile of dates $k_1$ miles, $D = 3000 - 5k_1$

Setting $3000 - 5k_1 = 2000$ implies $k_1 = 200$

Similarly, if $1000 < D \leq 2000$ the camel must make 2 trips to move the pile an additional $k_2$ miles, 1 round trip and 1 one-way trip. Thus after moving the pile the additional $k_2$ miles, $D = 2000 - 3k_2$

Setting $2000 - 3k_2 = 1000$ implies $k_2 = 333\frac{1}{3}$

Thus, the camel has 1000 dates remaining after moving the pile $533\frac{1}{3}$ miles.

During the remaining $466\frac{2}{3}$ miles the camel will eat 466 dates, leaving the number of dates delivered to market to be 534, with the camel being a little bit hungry.
Problem 2

Let 1 step per unit of time = professor’s walking rate
5 steps per unit of time = professor’s running rate
E steps per unit of time = escalator rate
D = the number of steps on the stopped escalator

It takes \( \frac{50}{1} = 50 \) units of time for the prof. to go down.
It takes \( \frac{125}{5} = 25 \) units of time for the prof. to go up.

Therefore \( 50(1 + E) = D \) and \( 25(5 - E) = D \)

So \( 50(1 + E) = 25(5 - E) \) implying that \( E = 1 \).

\[ \therefore D = 50(1 + 1) = 100 \]

There are 100 steps visible on the escalator when it is stopped.
Problem 3

For any polynomial function,

\[ f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x + a_0 \]

\[ f(1) = a_n + a_{n-1} + a_{n-2} + \cdots + a_1 + a_0 \], which is the sum of the coefficients.

Let \( f(x) = (5x - 2)^{16} \)

The sum of the coefficients is \( f(1) = (3)^{16} = 43046721 \)
Problem 4

At the end of 1 second the bug has crawled $\frac{1}{3}$ the length of the rope. Since the rope increases in length by 3 inches, during the next second the bug crawls $\frac{1}{6}$ the length of the rope. Similarly, during the next second the bug crawls $\frac{1}{9}$ the length of the rope, and so on.

We need to find $n$ such that

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{9} + \cdots + \frac{1}{3n} \geq 1$$

or

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \geq 3$$

Since the harmonic series diverges, it can be made as large as we wish. We find that if $n = 11$ the sum of the harmonic series is approximately 3.02.

Therefore the bug will reach the end of the rope a little before 11 seconds.
Problem 5

Let $x$, $y$ and $z$ be the measures of the angles. Without loss of generality let $0 < x \leq y \leq z$. We also know from geometry that $x + y + z = 180^\circ$. This equation and inequality give rise to the following inequalities:

\[
\begin{align*}
x &> 0 \\
y &\geq x \\
y &\leq -\frac{1}{2}x + 90
\end{align*}
\]

This system is graphed below.
The number of integer angled non-similar triangles is the number of lattice points in the figure. Zooming in on the intersection at the right side of the triangle gives this figure.

So the number of lattice points is:

\[
\frac{1 + 2 + 4 + 5 + 7 + \cdots + 88 + 89}{(1 + 2 + 3 + 4 + \cdots + 89) - (3 + 6 + 9 + \cdots + 87)} = \frac{89 \cdot 90}{2} - \frac{29 \cdot 90}{2} = \frac{60 \cdot 90}{2} = 2700
\]

(Note: this is also the area of the triangle. Why?)
Problem 6

Let \( n \) = the page number of the first missing page.

Then for some \( n > 0 \) and \( r > 0 \)

\[
\begin{align*}
n + (n + 1) + (n + 2) + \cdots + (n + r) & = 8656 \\
(r + 1)n + \frac{r(r + 1)}{2} & = 8656 \\
(r + 1)(2n + r) & = 17312 = 2^5 \cdot 541
\end{align*}
\]

If \( r \) is odd, \((r + 1)\) is even and \((2n + r)\) is odd. So \( r + 1 = 32 \) and \( 2n + r = 541 \). Solving the system gives \( n = 255 \) and \( r = 31 \). So the missing pages are from 255 to 286.

If \( r \) is even, \((r + 1)\) is odd and \((2n + r)\) is even. So \( r + 1 = 541 \) and \( 2n + r = 32 \). Solving the system gives \( n = -254 \) and \( r = 540 \). This contradicts the fact that \( n > 0 \) so there are no such page numbers.

Therefore the missing pages must be from 255 to 286, inclusive.
Problem 7

Consider the dice above:

\( \frac{2}{3} \) of the time A rolls a 4 and beats B.

\( \frac{2}{3} \) of the time C rolls a 2 and loses to B.

\( \frac{1}{3} \) of the time C rolls a 6 and beats D. The other \( \frac{2}{3} \) of the time C rolls a 2 and, \( \frac{1}{2} \) of that time D rolls a 1 and C wins. So C beats D \( \frac{1}{3} + \left( \frac{2}{3} \cdot \frac{1}{2} \right) = \frac{2}{3} \) of the time.

\( \frac{1}{2} \) of the time D rolls a 5 and beats A. The other \( \frac{1}{2} \) of the time D rolls a 1 and, \( \frac{1}{3} \) of that time A rolls a 0 and D wins. So D beats A \( \frac{1}{2} + \left( \frac{1}{2} \cdot \frac{1}{3} \right) = \frac{2}{3} \) of the time.

So A beats B, B beats C, C beats D and D beats A, all with a probability of \( \frac{2}{3} \).

Transitive? Not so much.
Problem 8

<table>
<thead>
<tr>
<th></th>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td># hits</td>
<td>$y - 3$</td>
<td>$y$</td>
</tr>
<tr>
<td># at bats</td>
<td>$x - 4$</td>
<td>$x$</td>
</tr>
<tr>
<td>Average</td>
<td>.233</td>
<td>.252</td>
</tr>
</tbody>
</table>

\[
.2325 \leq \frac{y - 3}{x - 4} < .2335 \quad \text{and} \quad .2515 \leq \frac{y}{x} < .2525
\]

Solving these inequalities for $x$ gives

\[
\frac{y - 2.066}{.2335} < x \leq \frac{y - 2.07}{.2325} \quad \text{and} \quad \frac{y}{.2525} < x \leq \frac{y}{.2515}
\]

Combining these inequalities gives

\[
\frac{y - 2.066}{.2335} < \frac{y}{.2515} \quad \text{and} \quad \frac{y}{.2525} < \frac{y - 2.07}{.2325}
\]

Solving gives $y < 28.86$ and $y > 26.13$. Since $y$ is a whole number $y = 27$ or $y = 28$.

If $y = 27$, then $111.07 < x \leq 111.5$, which can’t happen since $x$ is a natural number.

If $y = 28$, then $106.8 < x \leq 107.2$ and $106.9 < x \leq 107.3$, so $x = 107$

The player has been at bat 107 times after the game.
Problem 9

\[ \triangle OQS \] is a right triangle since it is inscribed in a semicircle. Hence \( QS = \sqrt{4 - x^2} \)

\( m \angle QPS = m \angle PQS \) since they are both complementary to congruent angles \( \angle QRO \) and \( \angle RQO \).

\( QS = SP \) since they are opposite equal angles.

\[ OP = OS + SP = OS + QS = 2 + \sqrt{4 - x^2} \]

So,

\[ \lim_{x \to 0} OP = \lim_{x \to 0} 2 + \sqrt{4 - x^2} = 4 \]
\Delta QSR \sim \Delta QOP \text{ so } \frac{OP}{x} = \frac{1 - \cos x}{x - \sin x}

\[ OP = \frac{x - x \cos x}{x - \sin x} \]

\[ \lim_{x \to 0} OP = \lim_{x \to 0} \frac{x - x \cos x}{x - \sin x} \]

Using L’Hôpital’s Rule repeatedly allows us to evaluate the limit.

\[ \lim_{x \to 0} \frac{x - x \cos x}{x - \sin x} = \lim_{x \to 0} \frac{1 + x \sin x - \cos x}{1 - \cos x} = \lim_{x \to 0} \frac{x \cos x + 2 \sin x}{\sin x} = \lim_{x \to 0} \frac{-x \sin x + 3 \cos x}{\cos x} = 3 \]