Success Center
Directed Learning Activity (DLA)

Adding & Subtracting Integers

M004.1
Directed Learning Activity – Adding and Subtracting Integers: A Money Model

Introduction: Although the rules for adding and subtracting integers are fairly simple, many students struggle with them. The purpose of this Directed Learning Activity (DLA) is to help you understand not only how to apply those rules, but also why those rules are what they are. There are many ways to add and subtract integers, and in this activity we will use your understanding of money to increase your comprehension of the relevant rules.

Note: You should not use a calculator at all when completing this DLA.

Prior Knowledge: In order to complete this activity, you need to know:

- The set of whole numbers: \{0, 1, 2, 3, \ldots\}
- The set of integers: \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}
- How to add and subtract whole numbers.

Step One: Introducing the Money Model

Suppose you have $10 and you spend $7. What is your balance? To find your balance you need to find the difference between the money you had and the money you spent. In this case, we would say your balance is $3 because $10 - $7 = $3.

Now suppose you have $7 in the bank and you spend $10 (by charging it to a credit card). What is your balance? Since you spent more money than what you had, you are now in debt. In this case you have a negative balance of $3. We say that your balance is \(-$3\). Notice that the difference between the money that you had and the money that you spent is still $3, but since you spent more than what you had, you have a negative balance.

In general, to find your balance when you have money and you spend money, you subtract the smaller amount from the larger amount. If you had more money than what you spent, you have a **positive balance**. If you spent more money than what you had, you have a **negative balance**.

Complete the following table:

<table>
<thead>
<tr>
<th>Amount You Have</th>
<th>Amount You Spend</th>
<th>Difference Larger Amount – Smaller Amount</th>
<th>Your Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$120</td>
<td>$85</td>
<td>120 – 85 =</td>
<td></td>
</tr>
<tr>
<td>$35</td>
<td>$57</td>
<td>57 – 35 =</td>
<td></td>
</tr>
</tbody>
</table>
Now, check your answers:

1. $35  
2. $22  
3. $36  
4. $232  
5. $7,877

If you missed any problems on the table, make sure you understand your mistake before continuing. If you need help, please see a tutor at this time.

**Step Two: Adding Two Integers**

Now, to develop our “money model,” you are going to think of positive integers as money you have; you will call them “paychecks.” You are going to think of negative integers as money you spend; you will call them “bills.” Now, when you are adding two integers, you just need to find your balance.

On the table above, you found the balance when you had one paycheck and one bill by finding their difference. That is, you subtracted. Why? Because it makes sense!

Now, if you had two paychecks and you wanted to know your balance, would you add them or subtract them? What if you had two bills? Think about it!

The idea here is not for you to memorize a bunch of rules, but to help you learn how to add integers in a way that makes sense to you.

**Example 1:** Suppose you want to do the following addition:

\[-8 + 5\]

Are there any paychecks? Yes. The 5 is positive, so we think of it as a $5 paycheck. Are there any bills? Yes. The 8 is negative, so we think of it as an $8 bill. What is your balance? Since you have a paycheck and a bill, you need to find the difference between them: \(8 - 5 = 3\). Since your bill is larger than your paycheck, you have a negative balance. Your balance is \(-$3\). Therefore:

\[-8 + 5 = -3\]

**Example 2:** Suppose you want to do the following addition:

\[-12 + (-13)\]
Are there any paychecks? No. There are no positive integers.
Are there any bills? Yes. The 12 and the 13 are negative, so you think of them as bills.
What is your balance? Since you have two bills, you add them to find out what is the total amount that you owe: \(12 + 13 = 25\). Since you are adding two bills, your balance is negative. Therefore:

\[-12 + (-13) = -25\]

Your turn: Study the four given examples and complete the following table.

<table>
<thead>
<tr>
<th>Find the Sum</th>
<th>Is Your “Balance” Positive or Negative? Explain.</th>
<th>To Find Your Balance, Should You Add or Subtract? Do It!</th>
<th>Final Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 22 + (−10)</td>
<td>Positive. My paycheck is larger than my bill.</td>
<td>Subtract. I have a paycheck and a bill. 22 − 10 = 12</td>
<td>12</td>
</tr>
<tr>
<td>B 46 + 31</td>
<td>Positive. I have two paychecks.</td>
<td>Add. 46 + 31 = 77</td>
<td>77</td>
</tr>
<tr>
<td>C −18 + (−37)</td>
<td>Negative. I have two bills.</td>
<td>Add. 18 + 37 = 55</td>
<td>−55</td>
</tr>
<tr>
<td>D 62 + (−89)</td>
<td>Negative. My bill is larger than my paycheck.</td>
<td>Subtract. I have a paycheck and a bill. 89 − 62 = 27</td>
<td>−27</td>
</tr>
<tr>
<td>1  −6 + 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2  −9 + (−6)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3  12 + 9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4  16 + (−7)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5  16 + (−19)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6  −54 + 54</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7  79 + (−51)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8  432 + 123</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9  −82 + 38</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 −654 + (−258)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now, check your answers:

1. −2  2. −15  3. 21  4. 9  5. −3  6. 0  7. 28  8. 555  9. −44  10. −912
If you missed any of the problems on the table, make sure you understand your mistake before continuing. If you need help, please see a tutor at this time.

Now, what have you learned? When you add two integers, you need to decide whether your answer will be positive or negative and whether you need to add or subtract. Just remember: think “paychecks” and “bills”!

**Step Three: Applying What You Have Learned**

Do the following additions. Show your work on this page and write your answers in the answer column.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $8 + (-15)$</td>
<td>2. $-4 + (-6)$</td>
<td>3. $-19 + 24$</td>
</tr>
<tr>
<td>4. $29 + 44$</td>
<td>5. $88 + (-59)$</td>
<td>6. $-47 + (-93)$</td>
</tr>
<tr>
<td>7. $126 + (-211)$</td>
<td>8. $-765 + (-640)$</td>
<td>9. $-870 + 870$</td>
</tr>
<tr>
<td>10. $-22,367 + 46,024$</td>
<td>11. $76,020 + (-79,511)$</td>
<td>12. $-11,875 + (-3,989)$</td>
</tr>
</tbody>
</table>

Answers

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
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<tr>
<td>2.</td>
<td></td>
<td></td>
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<tr>
<td>3.</td>
<td></td>
<td></td>
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<tr>
<td>4.</td>
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<td>5.</td>
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<tr>
<td>6.</td>
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<td>7.</td>
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<td>8.</td>
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<td>9.</td>
<td></td>
<td></td>
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<tr>
<td>10.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Step Four: Adding More Than Two Integers

Suppose now that you have to add a list of integers, such as:

\[3 + (-7) + (-8) + 5 + 8 + (-12) + 2\]

Using the money model you have been practicing, your approach should be to figure out how much money you had and how much money you spent. That is, you should first add up all your paychecks, then add up all your bills, and finally figure out your balance.

Let’s work on the example above. Begin by underlining all the paychecks:

\[\underline{3} + (-7) + \underline{5} + \underline{8} + (-12) + 2\]

Now, add all the underlined numbers to find the total amount you had: \[3 + 5 + 8 + 2 = 18\]

Next, add all the numbers that are not underlined to find the total amount you spent. Do not worry about the negative signs. They simply indicate that you have bills: \[7 + 8 + 12 = 27\]

So, the problem is simplified to finding your balance when you have $18 and you spend $27. That is:

\[18 + (-27)\]

This is the kind of problem just finished practicing. It should be easy for you to do.

\[18 + (-27) = -9\]

In short, when you have to add a list of integers, do the following: 1. add the paychecks, 2. add the bills, and 3. find the balance.

Your turn: Study the examples and complete the table.

<table>
<thead>
<tr>
<th>Original Expression</th>
<th>Simplified</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A [-12 + 9 + 6 + (-7) + 8 =]</td>
<td>[23 + (-19) =]</td>
<td>4</td>
</tr>
<tr>
<td>B [34 + (-43) + (-23) + 45 + (-21) =]</td>
<td>[79 + (-87) =]</td>
<td>-8</td>
</tr>
<tr>
<td>1 [5 + 11 + (-24) + 15 + (-8) =]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 [-4 + 9 + (-8) + (-5) + 2 + 3 + (-1) =]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 [-9 + 12 + (-15) + 8 + 4 =]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 [345 + (-673) + 98 + (-65) + (-342) =]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now, check your answers: 1. -1 2. -4 3. 0 4. -637
If you missed any of the problems on the table, make sure you understand your mistake before continuing. If you need help, please see a tutor at this time.

**Step Five: Applying What You Have Learned**

Do the following additions. Show your work on this page and write your answers in the answer column.

1. \(5 + (-8) + (-4) + 3 + 6\)  
2. \(-6 + (-6) + 4 + 8 + (-7)\)

3. \(15 + 8 + (-14) + 5 + (-16)\)  
4. \(-5 + (-28) + (-14) + (-18) + (-31)\)

5. \(65 + (-48) + (-74) + 93 + (-56)\)  
6. \(95 + (-86) + 124 + 73 + 61\)

7. \(-479 + 765 + 864 + (-962) + 228\)  
8. \(123 + (-345) + 567 + (-789) + 987 + (-765)\)
Step Six: Subtracting Integers

When mathematicians face a new problem, often they will try to restate it so that it looks like a problem they already know how to solve; this is exactly what you are going to do when you need to subtract integers. In fact, we define subtraction in terms of addition:

\[ a - b = a + (-b) \]

In other words, this equation states that “to subtract a number, you add its opposite.” This means that we can change every subtraction problem into an addition problem. Since you just learned how to add, you will have no problems with subtraction.

Study the following examples to learn how to rewrite a subtraction as an addition.

1. \(10 - 14 = 10 + (-14)\)  
   \([To\ subtract\ 14,\ you\ add\ (-14)]\)
2. \(9 - (-12) = 9 + 12\)  
   \([To\ subtract\ (-12),\ you\ add\ 12]\)
3. \(-2 - (-5) = -2 + 5\)  
   \([To\ subtract\ (-5),\ you\ add\ 5]\)
4. \(-3 - 8 = -3 + (-8)\)  
   \([To\ subtract\ 8,\ you\ add\ (-8)]\)

Your turn: Rewrite each subtraction as an addition. Do not compute the answer.

1. \(-12 - (-8) = \)  
2. \(-6 - 3 = \)  
3. \(15 - (-5) = \)  
4. \(2 - 8 = \)

Now, check your answers:

1. \(-12 + 8, \)  
2. \(-6 + 3, \)  
3. \(-15 + 5, \)  
4. \(4 + (-8)\)

If you missed any problem, make sure you understand your mistake before continuing. If you need help, please see a tutor at this time.

As you can see, if you know how to add integers and you know how to write a subtraction as an addition, then you will have no problem with subtraction of integers.

Step Seven: Applying What You Have Learned

Rewrite each subtraction as an addition, and then find the sum. The first one is done as an example.

1. \(-9 - (-12) = -9 + 12 = 3 \)  
2. \(-7 - 3 = \)  
3. \(6 - 11 = \)
4. \(12 - 8 = \)  
5. \(34 - (-45) = \)  
6. \(-92 - (-77) = \)
When you need to evaluate an expression that has several additions and subtractions, the first thing you should do is rewrite every subtraction in the expression as an addition. Carefully study the following examples, which show how to rewrite the expressions so that all subtractions are written as additions.

1. \[-3 + 5 - 7 + (-4) - (-3) = -3 + 5 + (-7) + (-4) + 3 = 8 + (-14) = -6\]
2. \[7 - 9 - (-3) - 12 + (-8) = -1 + (-3) + 4 + (-5)\]
3. \[-1 - (-2) - 3 - (-4) - 5 = -1 + 2 + (-3) + 4 + (-5) = 6 + (-9) = -3\]

After you have rewritten every subtraction as an addition, all you have to do is:

1. Add the paychecks, 2. add the bills, and 3. find the balance.

Let’s finish the three examples above. Notice that the paychecks are underlined in the second step, after all the subtractions have been rewritten as additions.

1. \[-3 + 5 - 7 + (-4) - (-3) = -3 + 5 + (-7) + (-4) + 3 = 8 + (-14) = -6\]
2. \[7 - 9 - (-3) - 12 + (-8) = -1 + (-3) + 4 + (-5)\]
3. \[-1 - (-2) - 3 - (-4) - 5 = -1 + 2 + (-3) + 4 + (-5) = 6 + (-9) = -3\]

Step Eight: Applying What You Have Learned

Rewrite each of the following expressions so that all subtractions are written as additions. Then, evaluate the expressions.

1. \[9 - (-1) + 2 - (-8)\]
2. \[-5 + 1 - 9 - (-7) + 6\]
3. \[23 - 144 - (87)\]
4. \[-98 + 125 - (-46) - 6\]
5. \[-2 - 3 - 4 - 5 - 4 - 3 - 2\]
6. \[-864 - (-921) - 48\]
7. \[30 + 27 - 14 + 11 - (-38) - 56\]
8. \[-84 - 48 - (-65) + 56 - 19 - (-91)\]
Step Nine: Reflection

a) Why did you (or your instructor) decide that completing this activity was a valuable learning experience?

b) Prior to completing this activity, did you struggle with addition or subtraction of integers? Please how these topics were presented in class. Be specific.

c) What was the most challenging part of this activity? How did you deal with this challenge?

d) Name something new you learned as a result of completing this activity that you think will help you do better in any of your classes. Be specific.

STOP. Please go over your work with a tutor at this time.
M004.1 – Adding and Subtracting Integers

PRINT STUDENT NAME                      STUDENT #

Tutor Feedback:

________ Student completed the entire activity.

________ Student demonstrated understanding of the process during the discussion of
his/her work.

________ Student was thoughtful in his reflections and expressed his/her thoughts using
complete sentences and proper grammar.

Additional Comments:

_____________________________________________________________________________________
_____________________________________________________________________________________
_____________________________________________________________________________________

PRINT INSTRUCTOR/TUTOR NAME                      DATE

INSTRUCTOR/TUTOR SIGNATURE

STUDENT – DO NOT FORGET TO TURN THIS SHEET IN
AT THE FRONT DESK!

You may not get credit for completing this DLA if you fail
to leave this sheet with the front desk receptionist.